NISTIR 6242

ANNUAL CONFERENCE ON FIRE RESEARCH
Book of Abstracts
November 2-5, 1998

Kellie Ann Beall, Editor

Building and Fire Research Laboratory
Gaithersburg, Maryland 20899

United States Department of Commerce
Technology Administration
National Institute of Standards and Technology

NISTIR 6242

ANNUAL CONFERENCE ON FIRE RESEARCH
Book of Abstracts
November 2-5, 1998

Kellie Ann Beall, Editor

October, 1998
Building and Fire Research Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899

U.S. Department of Commerce
William M. Daley, *Secretary*
Technology Administration
Gary Bachula, *Acting Under Secretary for Technology*
National Institute of Standards and Technology
Raymond G. Kammer, *Director*

THE DISCRETE-SOURCE APPROXIMATION OF RADIATIVE TRANSPORT IN FIRE PLUMES

Issam Lakkis and Ahmed F. Ghoniem
Massachusetts Institute of Technology

A novel approach for incorporating radiative transport processes in the simulation of fire plumes, or any other fluid dynamic/combustion process in which gaseous radiation plays an important role in determining the temperature field, has been developed. The approach is particularly suited for cases in which the radiating gas is confined within a relatively small subset of the overall flow domain, and when conditions deviate form those consistent with conventional thin, thick, or differential approximations of the radiative transport equations. Contrary to the methods of spherical harmonics and discrete ordinates, which results in a systems of PDEs that should be integrated, the proposed discrete source method is based on expressing the integrals in the radiative flux and the irradiation as sums of integrals over small volumes within which the radiating gas (source) temperature and properties can be assumed to be nearly uniform. The resulting integrals over these "discrete sources," each representing the radiation field of a single source, are then evaluated for target points which are close (near field) and distant (far field) from the source. In the case of almost constant absorption coefficient between the source and target, an equivalent to a radiative Biot-Savart law is obtained. The formulation can also be extended to the case of variable absorption coefficient by treating the exponent in the radiation kernels as a polynomial in the optical distance. The method has been validating by comparing its results for a number of cases to those obtained using direct solutions.

The formulation of the method starts with the following expression for the divergence of the radiative flux, which is the source terms in the energy equation, in terms of the black body radiation I_b and the incident radiation, or irradiation, G:

$$\nabla \cdot \vec{q} = \kappa(4\pi\, I_b - G) \tag{1}$$

where the κ is the linear absorption coefficient, $I_b = \dfrac{\sigma T^4}{\pi}$, σ being the Stefan-Boltzmann constant, and the irradiation, is;

$$G = \int_{4\pi} I\, d\Omega = \iiint_V \kappa(\vec{r}\,')\, I_b(\vec{r}\,')\, \frac{e^{-(\tau-\tau')}}{|\,\vec{r}-\vec{r}\,'\,|^2}\, dV(\vec{r}\,') + \iint_\Sigma I(\vec{r}\,'_\Sigma)\, \frac{e^{-\tau_\Sigma}}{|\,\vec{r}-\vec{r}\,'_\Sigma\,|^2}\, cos(\vec{r}-\vec{r}\,'_\Sigma, \vec{n}_\Sigma)\, d\Sigma(\vec{r}\,'_\Sigma) \tag{2}$$

where $\tau = \int_0^s \kappa\, ds$ is the optical path length, V is the volume of radiating gas and Σ is the surface surrounding this volume. A similar form can be written for the radiative flux. A similar expression can be written for the radiative flux.

Writing $T(\vec{x}) = \sum T_i\, f_\delta(\vec{x} - \vec{x}_i)$ where f is a symmetric function which is non zero only within a distance δ around x_i, the volume integrals in equations (2) for each single source can be evaluated as a generic source-target interaction (similar to the Biot-Savart law in hydrodynamics). The total irradiation is hence evaluated as the sum over all the elements' contributions. In the case of spatially uniform absorption coefficients, the integrals of the radiation kernel over the area of an individual source are evaluated via near field and far field expansions of the exponential term, e.g., for the near field:

$e^{-\kappa\sqrt{x}} \approx (1-\kappa\sqrt{x}) + \sum_{k=0}^{n} C_k \kappa^{2k} x^k$. It is possible then to obtain analytical solutions for the integrals, which can also be tabulated for computational efficiency. For instance, in the case of axisymmetric flow, the irradiation from a single element (ring) is given by: $G_i(\vec{r}) = \kappa(\vec{r}_i)\, I_b(\vec{r}_i)\ \Xi(\frac{R}{\delta}, \frac{\zeta}{\delta}, \frac{r}{\delta}, \kappa\delta)$ where R and r are the source and target radii, and ζ is the axial separation between them. The function Ξ pre-tabulated for the expected range of interest of all its arguments.

We have applied this approach to compute the irradiance and radiative flux from a square ring within which the temperature varies exponentially away from its center. The ring radius is taken to be unity, its width and height are 0.4. The direct solution was obtained by carrying out the triple integrals in equation (2) using a very fine mesh within and around the square ring to guarantee convergence. The discrete source solution was obtained by dividing up the square into 16 – 400 (4x4 – 20x20) elements. The comparison between the direct solution and the discrete source solution at different levels of refinement, for the normalized irradiation and flux are shown in figures 1 and 2, respectively. As shown in the figure, the method provides good accuracy even at relatively crude discretization of the body of radiating gas (since the near field of the sources is evaluated almost exactly). Extensions to: (1) the case of variable absorption coefficients, (2) the evaluation of radiative fluxes from a fire plume, and (3) the simulation of a fire plume with "internal" radiative transfer will be presented at the meeting.

REFERENCES:

1. Modest, M.F, *Radiative Heat Transfer*, McGraw Hill, 1993.
2. Ghoniem, A.F., Lakkis, I., and Soteriou, M, *Twenty Six (Int) Symposium on Combustion*, The Combustion Institute, Pittsburg, 1996, pp. 1531-1539.
3. Lakkis, I and Ghoniem, A.F. "Lagrangian Simulation of Fire Plumes," *7th AIAA/ASME Joint Thermophysics and Heat Transfer Conf.*, Albuquerque, NM, June 1998.