Brief Communication

The Elusive History of $\tilde{m} = 1.57 - 0.56i$ for the Refractive Index of Soot

KERMIT C. SMYTH* and CHRISTOPHER R. SHADDIX**
Building and Fire Research Laboratory, National Institute of Standards and Technology, Gaithersburg, Maryland 20899

Soot volume fractions, number densities, and particle sizes have been extensively measured in laboratory flames using combined extinction and scattering methods. In order to obtain quantitative results, a value of the complex refractive index of the soot must be chosen, appropriate for the wavelength employed in the experimental measurements. Recent quantitative soot volume fraction determinations using laser-induced incandescence [1–5] also rely on a proper calibration, which is usually based on extinction-derived soot concentrations. Soot refractive indices have been reported for over 30 years, during which time measurement techniques have evolved considerably. Extractive, room-temperature compressed pellet reflectance methods have been replaced, for example, by in situ determinations using combined classical and dynamic light scattering. Despite many measurements of the refractive index of soot (see below), $\tilde{m} = 1.57 - 0.56i$ is still by far the result most often cited in the combustion community for visible wavelengths. This value has been widely attributed to Dalzell and Sarofim [6], although it is not included in their experimental data in either the visible or near-IR wavelength regions.

Dalzell and Sarofim used a polarization ratio reflectance technique and found that $\tilde{m}$ was essentially constant over the wavelength range 435.8–806.5 nm with mean values of $1.56 - 0.46i$ and $1.57 - 0.50i$ for soot collected from acetylene and propane diffusion flames, respectively [6]. Measurements were also made in the region 2.5–10.0 μm, and a dispersion model was formulated for interpolating the results between 0.8 and 2.5 μm and extrapolating beyond the wavelength range of the experiments. In a subsequent paper, Dalzell et al. [7] reported $\tilde{m} = 1.60 - 0.60i$ for propane soot at 435.8 nm. Thus, for the laser sources typically employed in soot measurements (488.0 nm and 514.5 nm for argon-ion lasers and 632.8 nm for helium–neon lasers), there is no basis in either the Dalzell and Sarofim [6] or Dalzell et al. [7] work for the value of $\tilde{m} = 1.57 - 0.56i$.

D'Alessio et al. [8] were the first to use $\tilde{m} = 1.57 - 0.56i$ for the soot refractive index as "close to the value determined for soot produced by propane–air flames" by Dalzell et al. [7]. They chose this result in order to analyze their data from premixed CH_4/O_2 flames (Ref. 8, Figs. 3–5). It is noteworthy that these authors considered that this selection amounted to using the values of Dalzell and Sarofim (Ref. 8, p. 947), and indeed they asserted as much in two papers written soon thereafter. D'Alessio and Ambrosio [9] reviewed measurements of the soot refractive index and carried out an extensive analysis of the optical properties of soot with $\tilde{m} = 1.57 - 0.56i$ as the "average value of the determinations of Dalzell and Sarofim" [6] (the average of all their visible data is actually $\tilde{m} = 1.57 - 0.48i$). In the Fifteenth Symposium, D'Alessio et al. (including Sarofim as a coauthor [10]) referenced $\tilde{m} = 1.57 - 0.56i$ to Dalzell and Sarofim [6] as that "reported for propane and acetylene soots in the visible." This paper was subsequently

*Corresponding author.
**National Research Council NIST Postdoctoral Research Associate, 1993–1995; present address: Combustion Research Facility, Sandia National Laboratories, Livermore, CA 94551.

0010-2180/96/$15.00
PII S0010-2180(96)00170-8

COMBUSTION AND FLAME 107:314–320 (1996)
Copyright © 1996 by The Combustion Institute
Published by Elsevier Science Inc.

cited by other groups and seems to be the main source of the confusion: D'Alessio's soot investigations were sufficiently pre-eminent in the 1970s that most authors adopted his attribution of $\tilde{m} = 1.57 - 0.56i$ to Dalzell and Sarofim without question [3, 11–28]. In later work, the D'Alessio group both correctly cited [29] the experimental values of Dalzell and Sarofim [6] and continued to reference $\tilde{m} = 1.57 - 0.56i$ as coming from these authors [30]. This value for the soot refractive index has also been utilized without clearly indicating the source [31–33]. In addition, other values of the soot refractive index have been misattributed to Dalzell and Sarofim [6], specifically $\tilde{m} = 1.56 - 0.56i$ [34–36], $1.56 - 0.57i$ [37], $1.59 - 0.59i$ [38], $1.55 - 0.56i$ [39, 40], $1.5 - 0.5i$ [41], $1.57 + 0.56i$ [42–44], $1.56 - 0.54i$ [45]. Some of these references may involve typographical errors. A number of investigators, including us [2, 5], have used $\tilde{m} = 1.57 - 0.56i$ for ease of comparison with the results from other studies. Only in relatively few cases have the actual experimental data of Dalzell and Sarofim [6] been correctly cited [8, 46–66].

What effect does the choice of $\tilde{m} = 1.57 - 0.56i$ have on the derived soot volume fraction? The wide range of reported values for the refractive index of soot has been discussed recently by several investigators [30, 33, 62]. Figure 1 plots the results of experimental studies specifically designed to measure the soot

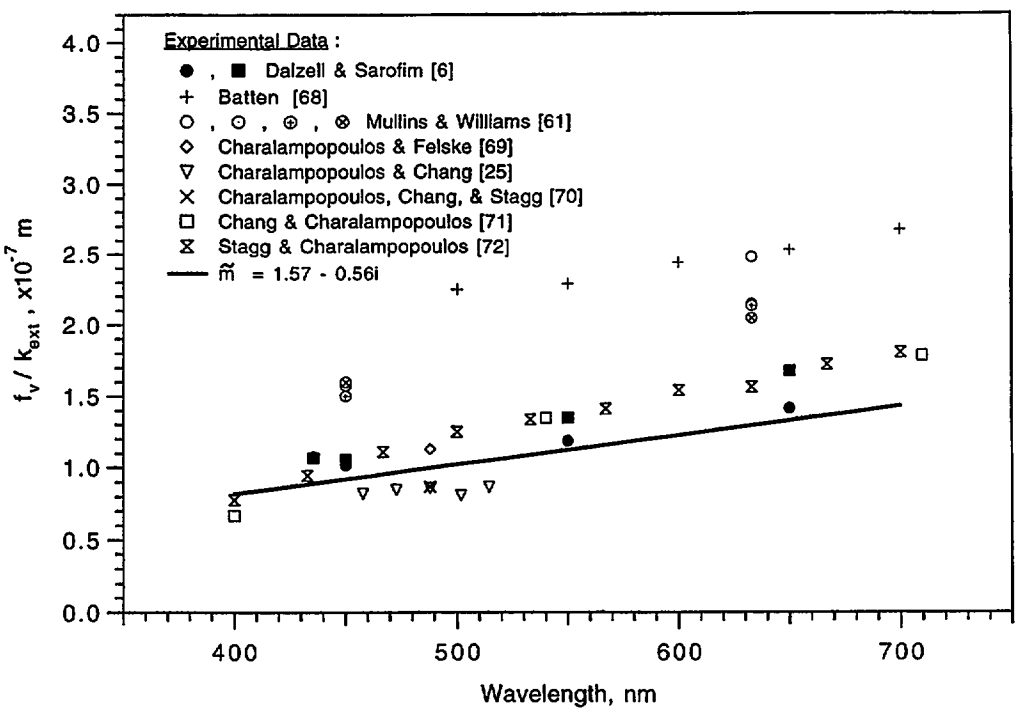

Fig. 1. Ratio of soot volume fraction to the extinction coefficient (in units of meters) for experimental values of the soot refractive index at visible wavelengths, compared to the result using $\tilde{m} = 1.57 - 0.56i$. The Dalzell and Sarofim [6] and Mullins and Williams [61] data are shown with multiple symbols in order to differentiate index of refraction determinations using different fuels. Data from Batten [68] are for vacuum-treated kerosene flame soot; the indices of refraction derived for the untreated kerosene soot give f_v/k_{ext} values which are nearly two times larger. The studies of Charalampopoulos and co-workers (with the exception of Stagg and Charalampopoulos [72]) involved in situ determinations of the index of refraction at a number of heights above premixed flat flames. Their data are presented here for the lowest reported height (generally 6 mm), at which the least extent of agglomeration has occurred. The Charalampopoulos, Chang, and Stagg [70] value is an average of the determinations for propane flames with three different equivalence ratios. Soot pellet reflectance results of Stagg and Charalampopoulos [72] are averaged over the three temperatures at which they were determined.

refractive index, and Fig. 2 presents the predictions of several semi-empirical expressions. In the Rayleigh limit, the ratio of the soot volume fraction, f_v, to the extinction coefficient, k_{ext}, is given by

$$\frac{f_v}{k_{ext}} = \frac{\lambda}{6\pi E(\tilde{m})}, \tag{1}$$

where

$$E(\tilde{m}) = -Im\left(\frac{\tilde{m}^2 - 1}{\tilde{m}^2 + 2}\right). \tag{2}$$

The extinction coefficient is usually determined from experiments as

$$k_{ext} = \frac{\ln\left(\frac{I_0}{I}\right)}{L}, \tag{3}$$

where I_0 and I are the incident and transmitted light intensities, respectively, and L is the pathlength through the soot. Figures 1 and 2 reveal that a substantial range of soot volume fractions can be derived from a given measured extinction. For example, at a laser wavelength near 632.8 nm (HeNe laser) the total variation of the f_v/k_{ext} ratio from the experimental measurements is 1.4×10^{-7} m to 2.5×10^{-7} m, or almost a factor of 2. To some extent, the measured range in $\tilde{m}$ at a given wavelength may reflect actual differences in the optical properties of the soot due to variations in soot temperature, C/H ratio, extent of agglomeration, and sample collection and preparation (for ex situ determinations). However, the experiments performed to measure the magnitudes of these effects [6, 25, 61, 67, 69–73] have generally found their influence to be much smaller than the variations in $\tilde{m}$

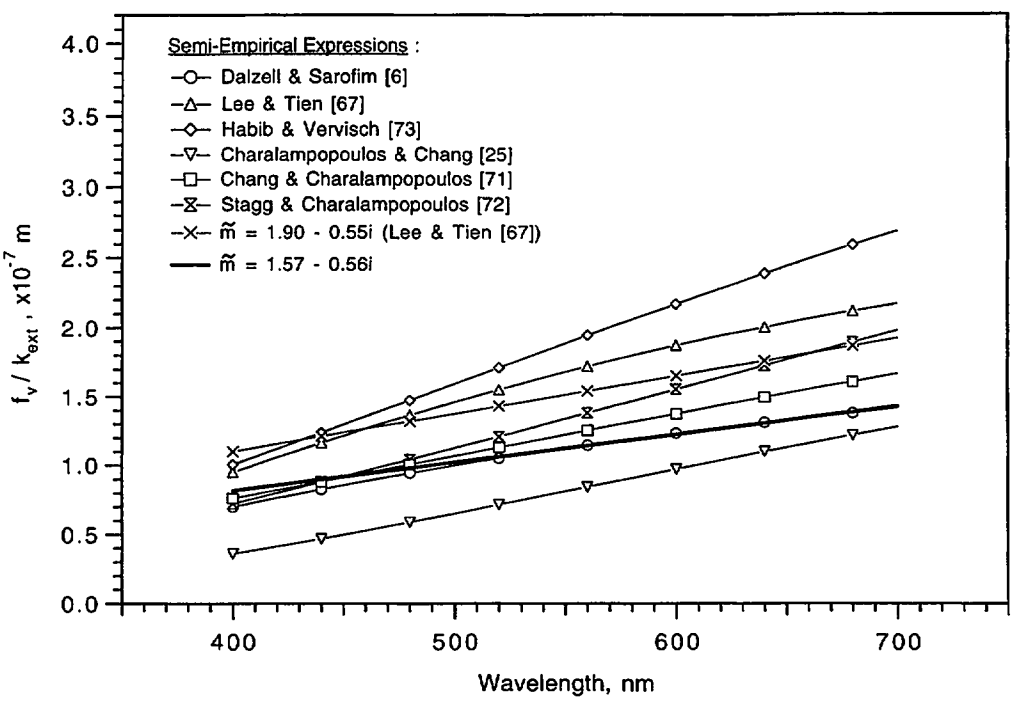

Fig. 2. Ratio of soot volume fraction to the extinction coefficient (in units of meters) for various semi-empirical expressions and the two most oft-cited indices of refraction at visible wavelengths. All of the curves except that of Chang and Charalampopoulos [71] and the fixed indices of refraction are determined from the Drude–Lorentz dispersion model of optical properties, with different values assumed for the electron number densities, frequencies, and damping constants. The spectral variation given by Chang and Charalampopoulos is described by Kramers–Krönig relations. Dispersion constants determined at 1450 K by Lee and Tien [67] are shown here.

(and f_v/k_{ext}) between different studies. Köylü and Faeth [74] have measured the ratio $F(\tilde{m})/E(\tilde{m})$ at 514.5 nm for several hydrocarbon fuels, where $F(\tilde{m})$ is given by

$$F(\tilde{m}) = \left|\frac{\tilde{m}^2 - 1}{\tilde{m}^2 + 2}\right|^2, \tag{4}$$

and found that their results (0.74 and 0.78) agree much better with that derived from $\tilde{m} = 1.57 - 0.56i$ (0.84) than when using more recent determinations for the soot refractive index. Note that this finding is also consistent with the actual experimental measurements of Dalzell and Sarofim [6], where $F(\tilde{m})/E(\tilde{m})$ is 0.84 and 0.83 for propane and acetylene soot, respectively, in the visible region.

Figures 1 and 2 show that $\tilde{m} = 1.57 - 0.56i$ yields a curve toward the low end of f_v/k_{ext} for visible wavelengths. In other words, the soot volume fraction derived from experimental extinction data using $\tilde{m} = 1.57 - 0.56i$ is lower than would be deduced from a soot refractive index based on most of the direct experimental measurements and the several proposed dispersion and Kramers–Krönig relations. Compared to the mean Dalzell and Sarofim values of $\tilde{m} = 1.56 - 0.46i$ and $\tilde{m} = 1.57 - 0.50i$ for acetylene and propane soot [6], the use of $\tilde{m} = 1.57 - 0.56i$ gives soot volume fractions which are low by 17% and 10%, respectively. The Lee and Tien result of $\tilde{m} = 1.90 - 0.55i$ [67] is also often cited in reporting soot volume fraction results (usually correctly). With this value of the refractive index, $E(\tilde{m})$ is 26% lower than for $\tilde{m} = 1.57 - 0.56i$, with a corresponding increase in the derived soot volume fraction.

Equation 1 is strictly valid only when the scattering contribution to the total light extinction is negligible, which is certainly not the case when using visible light sources with highly agglomerated soot aggregates (such as in the overfire region of turbulent diffusion flames [75 and references therein]). In these instances, measurements of the total scattering cross sections are required in addition to the total extinction in order to determine the amount of soot absorption. In practice, Eq. 1 is generally assumed to hold, and a light/laser source is chosen for extinction measurements which places the largest expected aggregate structures close to the Rayleigh limit. For many cases, the error resulting from the use of Eq. 1 is smaller than that suggested by the wide range of reported values for the soot refractive index [75, 76].

Based on the weight of experimental data (Fig. 1), the value of $\tilde{m} = 1.57 - 0.56i$ for the soot refractive index is likely to underestimate the soot volume fractions deduced from extinction measurements. However, as mentioned above, use of Eq. 1 to relate measured extinctions to soot volume fractions tends to overestimate the soot volume fraction. For particle sizes and agglomerate structures which yield an appreciable scattering contribution to the measured total extinction, using $\tilde{m} = 1.57 - 0.56i$ gives a soot volume fraction result closer to the true value. Nevertheless, such a simplified approach to data analysis may still lead to significant errors, since the degree to which scattering affects extinction measurements is typically unknown and will likely vary within a flame as the soot field evolves. In summary, future citation of $\tilde{m} = 1.57 - 0.56i$ may aid in comparing new results to previous analyses, but this value should not be used believing that it is founded on the measurements of Dalzell and Sarofim [6].

REFERENCES

1. Quay, B., Lee, T.-W., Ni, T., and Santoro, R. J., *Combust. Flame* 97:384–392 (1994).
2. Shaddix, C. R., Harrington, J. E., and Smyth, K. C., *Combust. Flame* 99:723–732 (1994) and 100:518 (1995).
3. Vander Wal, R. L., and Weiland, K. J., *Appl. Phys. B* 59:445–452 (1994).
4. Bengtsson, P.-E., and Aldén, M., *Appl. Phys. B* 60:51–59 (1995).
5. Shaddix, C. R., and Smyth, K. C., "Laser-Induced Incandescence Measurements of Soot Production in Steady and Flickering Methane, Propane, and Ethylene Diffusion Flames," *Combust. Flame* (in press).
6. Dalzell, W. H., and Sarofim, A. F., *J. Heat Transfer* 91:100–104 (1969).
7. Dalzell, W. H., Williams, G. C., and Hottel, H. C., *Combust. Flame* 14:161–169 (1970).
8. D'Alessio, A., Di Lorenzo, A., Beretta, F., and Venitozzi, C., *Fourteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1973, pp. 941–953.
9. D'Alessio, A., and Ambrosio, M., *La Termotecnica* 30:460–472 (1976, meeting held in 1974; in Italian).

10. D'Alessio, A., Di Lorenzo, A., Sarofim, A. F., Beretta, F., Masi, S., and Venitozzi, C., *Fifteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1975, pp. 1427–1438.
11. Haynes, B. S., Jander, H., and Wagner, H. Gg., *Seventeenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1979, pp. 1365–1374; Haynes, B. S., and Wagner, H. Gg., *Ber. Bunsenges. Phys. Chem.* 84:499–506 (1980); Haynes, B. S., Jander, H., and Wagner, H. Gg., *Ber. Bunsenges. Phys. Chem.* 84:585–592 (1980); Kent, J. H., Jander, H., and Wagner, H. Gg., *Eighteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1981, pp. 1117–1126; Baumgärtner, L., Jander, H., and Wagner, H. Gg., *Ber. Bunsenges. Phys. Chem.* 87:1077–1080 (1983); Baumgärtner, L., Hesse, D., Jander, H., and Wagner, H. Gg., *Twentieth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1984, pp. 959–967; Brei, M., Jander, H., and Wagner, H. Gg., *Ber. Bunsenges. Phys. Chem.* 91:30–36 (1987); Böhm, H., Hesse, D., Jander, H., Lüers, B., Pietscher, J., Wagner, H. Gg., and Weiss, M., *Twenty-Second Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1988, pp. 403–411.
12. Jagoda, I. J., Prado, G., and Lahaye, J., *Combust. Flame* 37:261–274 (1980); Prado, G., Jagoda, J., Neoh, K., and Lahaye, J., *Eighteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1981, pp. 1127–1136; Garo, A., Lahaye, J., and Prado, G., *Twenty-First Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1986, pp. 1023–1031.
13. Nakamura, M., Koda, S., and Akita, K., *Nineteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1982, pp. 1395–1401.
14. Bockhorn, H., Fetting, F., Wannemacher, G., and Wenz, H. W., *Nineteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1982, pp. 1413–1420; Bockhorn, H., Fetting, F., Heddrich, A., and Wannemacher, G., *Twentieth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1984, pp. 979–988; Bockhorn, H., Fetting, F., Heddrich, A., Meyer, U., and Wannemacher, G., *J. Aerosol Sci.* 19:591–602 (1988); Wieschnowsky, U., Bockhorn, H., and Fetting, F., *Twenty-Second Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1988, pp. 343–352; Bockhorn, H., and Schäfer, Th., in *Soot Formation in Combustion: Mechanisms and Models*, Springer Series in Chemical Physics 59 (H. Bockhorn, Ed.), Springer-Verlag, Berlin, (1994), pp. 253–274; Schäfer, Th., Mauß, F., Bockhorn, H., and Fetting, F., *Z. Naturforsch.* 50a:1009–1022 (1995).
15. Santoro, R. J., Semerjian, H. G., and Dobbins, R. A., *Combust. Flame* 51:203–218 (1983); Santoro, R. J., and Semerjian, H. G., *Twentieth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1984, pp. 997–1006; Dobbins, R. A., Santoro, R. J., and Semerjian, H. G., *Twenty-Third Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1990, pp. 1525–1532; Puri, R., Richardson, T. F., Santoro, R. J., and Dobbins, R. A., *Combust. Flame* 92:320–333 (1993); Quay, B., Lee, T.-W., Ni, T., and Santoro, R. J., *Combust. Flame* 97:384–392 (1994); Ni, T., Gupta, S. B., and Santoro, R. J., *Twenty-Fifth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1994, pp. 585–592.
16. Flower, W. L., *Phys. Rev. Lett.* 51:2287–2290 (1983); Flower, W. L., and Bowman, C. T., *Twentieth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1984, pp. 1035–1044; Flower, W. L., and Bowman, C. T., *Twenty-First Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1986, pp. 1115–1124: Flower, W. L., *Twenty-Second Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1988, pp. 425–435; Axelbaum, R. L., Flower, W. L., and Law, C. K., *Combust. Sci. Technol.* 61:51–73 (1988).
17. Harris, S. J., and Weiner, A. M., *Combust. Sci. Technol.* 32:267–275 (1983); 38:75–87 (1984); Harris, S. J., Weiner, A. M., and Ashcraft, C. C., *Combust. Flame* 64:65–81 (1986); Harris, S. J., and Weiner, A. M., *Twenty-Second Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1988, pp. 333–342.
18. Kent, J. H., and Bastin, S. J., *Combust. Flame* 56:29–42 (1984).
19. Vandsburger, U., Kennedy, I. M., and Glassman, I., *Twentieth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1984, pp. 1105–1112; Vandsburger, U., Kennedy, I., and Glassman, I., *Combust. Sci. Technol.* 39:263–285 (1984); Hura, H. S., and Glassman, I., *Combust. Sci. Technol.* 53:1–21 (1987); Hura, H. S., and Glassman, I., *Twenty-Second Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1988, pp. 371–378.
20. Dasch, C. J., *Appl. Opt.* 23:2209–2215 (1984).
21. Felske, J. D., Hsu, P.-F., and Ku, J. C., *J. Quant. Spectrosc. Radiat. Transfer* 6:447–465 (1986).
22. Andreussi, P., Barbieri, B., and Petarca, L., *Combust. Sci. Technol.* 49:123–141 (1986).
23. Weill, M. E., Lhuissier, N., and Gouesbet, G., *Appl. Opt.* 25:1676–1683 (1986); Lhuissier, N., Gouesbet, G., and Weill, M. E., *Combust. Sci. Technol.* 67:17–36 (1989).
24. Lawton, S. A., *Combust. Sci. Technol.* 57:163–169 (1988); *Combust. Flame* 75:175–181 (1989).
25. Charalampopoulos, T. T., and Chang, H., *Combust. Sci. Technol.* 59:401–421 (1988).
26. Zhang, C., Atreya, A., and Lee, K., *Twenty-Fourth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1992, pp. 1049–1057.
27. Coppalle, A., and Joyeux, D., *Combust. Sci. Technol.* 93:375–386 (1993).
28. Frenklach, M., and Wang, H., in *Soot Formation in Combustion: Mechanisms and Models*, Springer Series in Chemical Physics 59 (H. Bockhorn, Ed.), Springer-Verlag, Berlin, (1994), pp. 165–192.

29. Menna, P., and D'Alessio, A., *Nineteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1982, pp. 1421–1428.
30. Vaglieco, B. M., Beretta, F., and D'Alessio, A., *Combust. Flame* 79:259–271 (1990).
31. Neoh, K. G., Howard, J. B., and Sarofim, A. F., *Twentieth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1984, pp. 951–957.
32. Dasch, C. J., and Heffelfinger, D. M., *Combust. Flame* 85:389–402 (1991).
33. Choi, M. Y., Hamins, A, Mulholland, G. W., and Kashiwagi, T., *Combust. Flame* 99:174–186 (1994).
34. D'Alessio, A., in *Particulate Carbon: Formation During Combustion* (D. C. Siegla and G. W. Smith, Eds.), Plenum, New York, 1981, pp. 207–256.
35. Gomez, A., and Glassman, I., *Twenty-First Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1986, pp. 1087–1095; Gomez, A., Littman, M. G., and Glassman, I., *Combust. Flame* 70:225–241 (1987).
36. Scrivner, S. M., Taylor, T. W., Sorensen, C. M., and Merklin, J. F., *Appl. Opt.* 25:291–297 (1986); Sorensen, C. M., Cai, J., and Lu, N., *Appl. Opt.* 31:6547–6557 (1992); Cai, J., Lu, N., and Sorensen, C. M., *Langmuir* 9:2861–2867 (1993);
37. King, G. B., Sorensen, C. M., Lester, T. W., and Merklin, J. F., *Phys. Rev. Lett.* 50:1125–1128 (1983).
38. Bockhorn, H., Fetting, F., Heddrich, A., and Wannemacher, G., *Ber. Bunsenges. Phys. Chem.* 91:819–825 (1987).
39. Gore, J. P., and Faeth, G. M., *Twenty-First Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1986, pp. 1521–1531; Gore, J. P., and Faeth, G. M., *J. Heat Transfer* 110:173–181 (1988); Sivathanu, Y. R., Gore, J. P., and Faeth, G. M., *Combust. Flame* 73:315–329 (1988); Sivathanu, Y. R., and Faeth, G. M., *Combust. Flame* 81:133–149 and 150–165 (1990); Sivathanu, Y. R., Kounalakis, M. E., and Faeth, G. M., *Twenty-Third Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1990, pp. 1543–1550; Köylü, Ü. Ö., Sivathanu, Y. R., and Faeth, G. M., *Fire Safety Science—Proc. Third International Symposium*, pp. 625–634 (1991); Köylü, Ü. Ö., and Faeth, G. M., *Combust., Flame* 87:61–76 (1991) and 89:140–156 (1992); Sunderland, P. B., Köylü, Ü. Ö., and Faeth, G. M., *Combust. Flame* 100:310–322 (1995).
40. Sivathanu, Y. R., Gore, J. P., and Dolinar, J., *Combust. Sci. Technol.* 76:45–66 (1991); Sivathanu, Y. R., and Gore, J. P., *Combust. Sci. Technol.* 80:1–21 (1991); Gore, J., Klassen, M., Hamins, A., and Kashiwagi, T., *Fire Safety Science—Proc. Third International Symposium*, pp. 395–404 (1991); Klassen, M., Sivathanu, Y. R., and Gore, J. P., *Combust. Flame* 90:34–44 (1992); Klassen, M., Gore, J. P., Sivathanu, Y. R., Hamins, A., and Kashiwagi, T., *Twenty-Fourth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1992, pp. 1713–1719.
41. Selamet, A., and Arpaci, V. S., *Int. J. Heat Mass Transfer* 32:1809–1820 (1989); Selamet, A., and Arpaci, V. S., *Combust. Sci. Technol.* 78:165–175 (1991); Al-Nimr, M. A., and Arpaci, V. S., *J. Heat Transfer* 114:950–957 (1992).
42. Bonczyk, P. A., and Hall, R. J., *Langmuir* 7:1274–1280 (1991).
43. Li, S. C., *Combust. Sci. Technol.* 77:149–169 (1991).
44. Köylü, Ü. Ö., and Faeth, G. M., *J. Heat Transfer* 115:409–417 (1993); 116:152–159 (1994); 116:971–979 (1994) citing Ref. 7; Faeth, G. M., and Köylü, Ü. Ö., *Combust. Sci. Technol.* 108:207–229 (1995); Lin, K.-C., and Faeth, G. M., *J. Propulsion and Power* 12:10–17 (1996); Sunderland, P. B., and Faeth, G. M., *Combust. Flame* 105:132–146 (1996).
45. Tree, D. R., and Foster, D. E., *Combust. Sci. Technol.* 95:313–331 (1994).
46. Lowes, T. M., and Newall, A. J., *Combust. Flame* 16:191–194 (1971).
47. Radcliffe, S. W., and Appleton, J. P., *Combust. Sci. Technol.* 3:255–262 (1971); Park, C., and Appleton, J. P., *Combust. Flame* 20:369–379 (1973).
48. Feugier, A., *Combust. Flame* 19:249–256 (1972).
49. Graham, S. C., *Combust. Sci. Technol.* 9:159–163 (1974); Graham, S. C., Homer, J. B., and Rosenfeld, J. L. J., *Proc. Roy. Soc. London A* 344: 259–285 (1975); Graham, S. C., *Proc. Roy. Soc. London A* 377:119–145 (1981).
50. Eckbreth, A. C., *J. Appl. Phys.* 48:4473–4479 (1977); values given are for acetylene soot, not propane.
51. Penner, S. S., and Chang, P., *J. Quant. Spectrosc. Radiat. Transfer* 20:447–460 (1978); Bleiweiss, M. C., Chang, P. H. P., and Penner, S. S., *J. Quant. Spectrosc. Radiat. Transfer* 26:273–275 (1981).
52. Delfau, J. L., Michaud, P., and Barassin, A., *Combust. Sci. Technol.* 20:165–177 (1979).
53. Bonczyk, P. A., *Combust. Flame* 35:191–206 (1979); Bonczyk, P. A., and Sangiovanni, J. J., *Combust. Sci. Technol.* 36:135–147 (1984).
54. Cashdollar, K. L., Lee, C. K., and Singer, J. M., *Appl. Opt.* 18:1763–1769 (1979).
55. Pagni, P. J., and Bard, S., *Seventeenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1979, pp. 1017–1028.
56. Pluchino, A. B., Goldberg, S. S., Dowling, J. M., and Randall, C. M., *Appl. Opt.* 19:3370–3372 (1980).
57. Haynes, B. S., and Wagner, H. Gg., *Prog. Energy Combust. Sci.* 7:229–273 (1981).
58. Siegel, R., and Howell, J. R., *Thermal Radiation Heat Transfer*, 2nd Ed., ch. 17, Hemisphere Publishing, New York, 1981.
59. Hirleman, E. D., and Moon, H. K., *J. Colloid Interface Sci.* 87:124–139 (1982).
60. Nishida, O., and Mukōhara, S., *Combust. Sci. Technol.* 35:157–173 (1983).
61. Mullins, J., and Williams, A., *Fuel* 66:277–280 (1987).
62. Simmons, B., and Williams, A., *Combust. Flame* 71:219–232 (1988).
63. Grosshandler, W. L., *Combust. Flame* 78:257–262 (1989).

64. Bengtsson, P.-E., and Aldén, M., *Appl. Phys. B* 48:155–164 (1989).
65. Kamimoto, T., Bae, M.-H., and Kobayashi, H., *Combust. Flame* 75:221–228 (1989).
66. Dobbins, R. A., Mulholland, G. W., and Bryner, N. P., *Atmos. Environ.* 28:889–897 (1994).
67. Lee, S. C., and Tien, C. L., *Eighteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1981, pp. 1159–1166.
68. Batten, C. E., *Appl. Opt.* 24:1193–1199 (1985).
69. Charalampopoulos, T. T., and Felske, J. D., *Combust. Flame* 68:283–294 (1987).
70. Charalampopoulos, T. T., Chang, H., and Stagg, B., *Fuel* 68:1173–1179 (1989).
71. Chang, H., and Charalampopoulos, T. T., *Proc. Roy. Soc. London A* 430:577–591 (1990).
72. Stagg, B. J., and Charalampopoulos, T. T., *Combust. Flame* 94:381–396 (1993).
73. Habib, Z. G., and Vervisch, P., *Combust. Sci. Technol.* 59:261–274 (1988).
74. Köylü, Ü. Ö., and Faeth, G. M., *J. Heat Transfer* 116:152–159 (1994); 118:415–421 (1996).
75. Faeth, G. M., and Köylü, Ü. Ö., *Combust. Sci. Technol.* 108:207–229 (1995).
76. Köylü, Ü. Ö., and Faeth, G. M., *J. Heat Transfer* 116:971–979 (1994).

Received 7 December 1995; revised 16 August 1996