

# **EVALUATION OF APPROXIMATE MODELS OF BUOYANT TURBULENT FLOWS**

by

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# EVALUATION OF APPROXIMATE MODELS OF BUOYANT TURBULENT FLOWS

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**Introduction.** At the present time, it is necessary to use approximate turbulence models in order to analyze the properties of practical fires due to the computational intractability of fully resolved three-dimensional time-dependent numerical simulations of buoyant turbulent flows representative of fire environments. Developing reliable models to treat buoyancy/turbulence interactions, however, has been inhibited due to the absence of measurements needed to evaluate both model approximations and predictions. Thus, the main objective of the present investigation was to compete measurements of the mean and turbulent properties of a classical buoyant turbulent flow that is frequently used to evaluate the predictions of turbulence models; namely, the round buoyant turbulent plume in the fully-developed (self-preserving) region far from the source. The new measurements also are used to initiate evaluation of turbulence modeling ideas, considering both classical similarity concepts [1-4], and turbulence models of varying complexity, e.g. [5-7]. The following description of the study is brief, more details and a complete summary of data can be found in [8-10].

There have been numerous past experimental studies of round buoyant turbulent plumes, see [1-18], and references cited therein. Nevertheless, new measurements are needed in order to address two limitations of past work; namely, concerns about whether past studies had actually achieved observations in the self-preserving region, and the need for more complete information about turbulence quantities within the self-preserving region. Discussion of whether past work had achieved self-preserving conditions can be found in Dai et al. [8-10]; the main question was that these measurements were generally confined to  $(x-x_0)/d \leq 62$ , which seems marginal based on observations of self-preserving conditions for nonbuoyant round turbulent jets [19,20]. In addition, available measurements of turbulence quantities generally were confined to lower-order moments which are only indirectly helpful for developing a better understanding of buoyancy/turbulence interactions.

**Experimental Methods.** Plume conditions were simulated using a source flow of dense gases (carbon dioxide and sulfur hexafluoride) in still air within a screened enclosure. Mean and fluctuating velocities were measured using laser velocimetry (LV). Scalar properties were represented by the mixture fraction (the mass fraction of source gas in a sample) using state relationships for isothermal mixing of ideal gases to find other scalars from the mixture fraction. Measurements of mixture fractions involved seeding the source flow with iodine vapor and using laser-induced iodine fluorescence (LIF). The LV and LIF measurements also allowed determination of velocity/mixture fraction statistics, as discussed in [21].

Experimental uncertainties of the present measurements, as well as various conservation checks of the measurements, are discussed by Dai et al. [8-10]. The evaluations showed that the measurements satisfied the governing equations, and that the integral forms of buoyancy and momentum fluxes were all satisfied, within experimental uncertainties. Finally, direct measurements showed that ambient velocities were properly negligible, and that doubling the removal rate of plume gases by the exhaust system

(from the normal removal rate) had a negligible effect on the distribution of mean mixture fractions in the self-preserving region of the flow.

**Results and Discussion.** Present measurements of mean mixture fractions are plotted in Fig. 1. These results are plotted in terms of scaled variables so that the measurements should approach the universal function,  $F(r/(x-x_0))$ , within the self-preserving region of the flow. As distance from the source progressively increases, the profiles of  $F$  become progressively narrower, with larger values near the axis; nevertheless, the measurements become independent of distance from the source for  $(x-x_0)/d \geq 87$  for the two plume sources, yielding proper self-preserving behavior. Similar behavior was observed for  $U(r/(x-x_0))$ , the scaled mean streamwise velocity function for the self-preserving region [9]. The tendency for the flow to become progressively narrower as transitional plumes develop toward self-preserving behavior also can be seen from past measurements of characteristic plume radii summarized in Table 1. The results of Papantoniou and List [11], which were obtained at similar distances from the source as present work, tend to confirm present measurements, while other measurements progressively nearer to the sources yield progressively larger scaled flow widths similar to the behavior of the present plumes seen in Fig. 1.

Table 1. Characteristic plume radii measurements

Source	Present	[11]	[12,13]	[14,15,17]
$(x-x_0)/d^a$	87-151	105	12-62	8-25
$l_w/(x-x_0)$	0.10	—	0.11	0.13-0.14
$l_f/(x-x_0)$	0.09	0.09	0.11	0.12

<sup>a</sup>Range of streamwise distances used to find self-preserving properties.

The differences between past and present estimates of self-preserving turbulent plume properties have a considerable impact on the evaluation of models of turbulence. This can be illustrated based on results recently reported by Pivovarov et al. [6]. This study involved testing a simplified contemporary turbulence model for self-preserving conditions, and evaluating model predictions based on the measurements of [14-18] — all of which involve transitional plumes based on present findings. Pivovarov et al. [6] should be consulted for the details, however, the model involved the k-ε-g approach that is widely used in field models of flames [5]. The main parameters that were adjusted were the empirical constant  $C_\mu = \nu_T \epsilon / k^2$ , and the effective turbulence Prandtl/Schmidt number,  $\sigma_T$ . All other model constants can be found in [5]. Present results involved repeating calculations and the evaluation of Pivovarov et al [6].

Measured and predicted values of  $U$  and  $F$  for self-preserving conditions are illustrated in Figs. 2 and 3, respectively. Present predictions and those of [6] are in good agreement for the same values of the turbulence model constants, and all predictions are not very sensitive to changes of  $\sigma_T$  between 0.7 and 0.9. However, increasing  $C_\mu$  causes the profiles to become broader. Thus, Pivovarov et al. [6] recommend larger values of  $C_\mu$  than the widely accepted value,  $C_\mu = 0.09$ , because this provided the best agreement

with the transitional plume measurements of [14-18] that they considered. In contrast, use of  $C_\mu = 0.09$ , yields results that are in good agreement with the present measurements.

In order to gain insight concerning present predictions and measurements of  $U$  and  $F$ , present predictions of the related turbulence quantities,  $k$  and  $\bar{f}'$ , in the self-preserving region are illustrated in Figs. 4 and 5. These results show that  $k$  and  $\bar{f}'$  exhibit self preserving behavior for  $(x-x_0)/d \geq 87$ ; this behavior was observed for all measurements of turbulence quantities for present flows. In addition, plots of  $k$  are nearly the same for self-preserving round nonbuoyant turbulent jets [7] and buoyant turbulent plumes [9], suggesting modest effects of buoyancy/turbulence interactions for this property. Finally, predictions of  $k$  using the standard turbulence model constants are seen to be in excellent agreement with present measurements.

The behavior of  $\bar{f}'/\bar{f}_c$  in Fig. 5 exhibits some interesting differences from the analogous velocity fluctuation properties,  $k^{1/2}/\bar{u}_c$ , in Fig. 4 for self-preserving buoyant turbulent plumes. The most significant difference is that  $k^{1/2}/\bar{u}_c$  and  $\bar{f}'/\bar{f}_c$  are similar for self-preserving nonbuoyant turbulent jets but are rather different for buoyant turbulent plumes. In particular,  $\bar{f}'/\bar{f}_c \approx 0.45$  but  $k^{1/2}/\bar{u}_c \approx 0.24$  for buoyant turbulent plumes while  $\bar{f}'/\bar{f}_c \approx k^{1/2}/\bar{u}_c \approx 0.24$  for nonbuoyant turbulent jets while the dip near the axis seen for  $k^{1/2}/\bar{u}_c$  for both flows, and for  $\bar{f}'/\bar{f}_c$  for nonbuoyant turbulent jets, is absent for buoyant turbulent plumes. The approximate turbulence model using standard model constants, however, correctly represents this behavior although predictions tend to overestimate measurements of  $\bar{f}'$  significantly near the edge of the flow. In contrast, predictions based on a lower value of  $C_{g2}$  used in some models, see [6], substantially overestimates  $\bar{f}'$  over the entire cross section of the flow.

Insight concerning the differences between the behavior of  $k^{1/2}/\bar{u}_c$  and  $\bar{f}'/\bar{f}_c$  for self-preserving round buoyant turbulent plumes can be obtained by considering the budgets for these quantities illustrated in Figs. 6 and 7. The most distinctive differences between the budgets for the two variables involve production: notably radial and total production are nearly the same for  $k/\bar{u}_c^2$  but total production is significantly larger than radial production for  $\bar{f}'/\bar{f}_c$  near the axis. Thus, it is the enhanced production due to streamwise effects that cause  $\bar{f}'/\bar{f}_c$  to be significantly larger than  $k^{1/2}/\bar{u}_c$  near the axis of buoyant turbulent plumes. Physically, this behavior comes about due to the streamwise instability of plumes, i.e., the density always approaches the ambient density with increasing distance along the axis, so that the flow is always prone to convective instability. The potential for instability is particularly strong for round buoyant turbulent plumes due to the unusually rapid decay of mean mixture fractions (and thus mean density) defect in the streamwise direction, e.g.,  $\bar{f}_c \sim (x-x_0)^{-5/3}$  for self-preserving plumes. While it is encouraging that the simplified turbulence model appears to account for this effect, it would be very interesting to evaluate whether the simplified predictions continue to be successful for line plumes where the variation of  $\bar{f}'$  in the streamwise direction is much slower, e.g.,  $\bar{f}' \sim (x-x_0)^{-1}$ .

In spite of the promising predictions of the simplified model seen in Figs. 2-5 for self-preserving buoyant turbulent plumes, however, there are several significant deficiencies that motivate consideration of higher-order turbulence models [10]. First of all, while the radial turbulent transport of mass and momentum properly satisfy the gradient diffusion approximation, the streamwise turbulent transport of mass and momentum do not near the edge of the flow. Another difficulty of the simplified model involves the gradient diffusion hypothesis with constant turbulent Prandtl/Schmidt numbers. The approximation is even problematical for transport in the radial direction within self-preserving round buoyant turbulent plumes. This behavior is evident from

present measurements of  $\sigma_T$  plotted in Fig. 8. It is clear that the assumption of a constant  $\sigma_T = 0.7$  across the width of the flow, typical of simplified turbulence models, is not supported by the measurements.

Thus, in spite of some promising predictions of the simplified turbulence model considered in Refs. 5 and 6, the present evaluation suggests significant concerns about applying these methods to complex buoyant turbulent flows in fire environments, where both streamwise and cross stream gradients frequently are comparable. It is hoped that the extensive measurements of higher-order turbulence quantities, available in Refs. 7-9, will prove helpful for developing more reliable methods.

**Nomenclature.**  $B_0$  = source buoyancy flux,  $C_\mu$  and  $C_{\epsilon 2}$  = turbulence modeling constants,  $d$  = source diameter,  $f$  = mixture fraction,  $F$  = normalized radial distribution of  $\bar{f}$ ,  $g$  = acceleration of gravity or  $\bar{f}''^2$ ,  $k$  = turbulence kinetic energy,  $l_f$  and  $l_u$  = characteristic plume radii based on  $\bar{f}$  and  $\bar{u}$ ,  $r$  = radial distance,  $u$  = streamwise velocity,  $U$  = normalized radial distribution of  $\bar{u}$ ,  $x$  = streamwise distance,  $\epsilon$  = rate of dissipation of turbulence kinetic energy,  $\nu_T$  = effective turbulence kinematic viscosity,  $\rho$  = density,  $\sigma_T$  = effective turbulence Prandtl/Schmidt number; subscripts:  $c$  = centerline value,  $o$  = initial value or virtual origin location,  $\infty$  = ambient value; superscripts:  $(\bar{\quad})$  = mean value,  $(\quad)'$  = rms fluctuating value.

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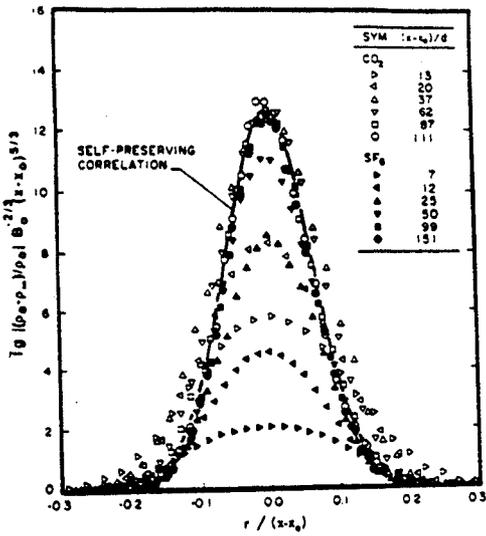


Fig 1. Development of radial profiles of mean mixture fractions.

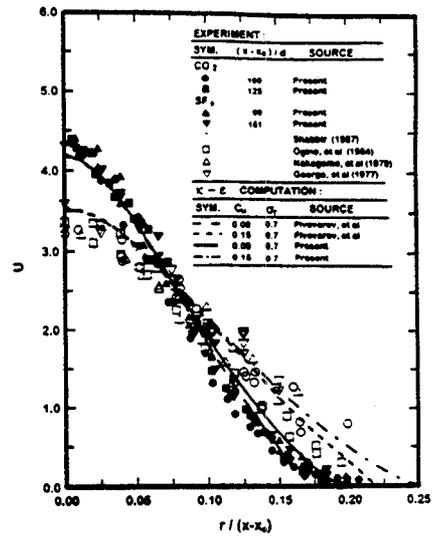


Fig 2. Measured and predicted mean streamwise velocity distributions in the self-preserving region

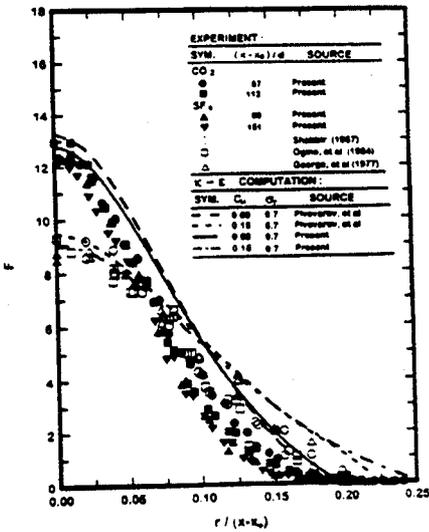


Fig 3. Measured and predicted mean mixture fraction distribution in the self-preserving region.

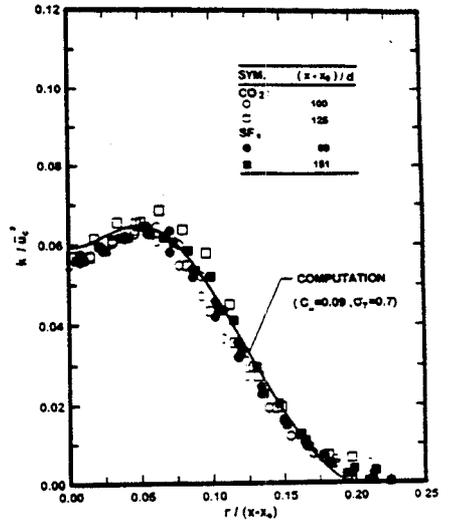


Fig 4. Measured and predicted turbulence kinetic energy in the self-preserving region.

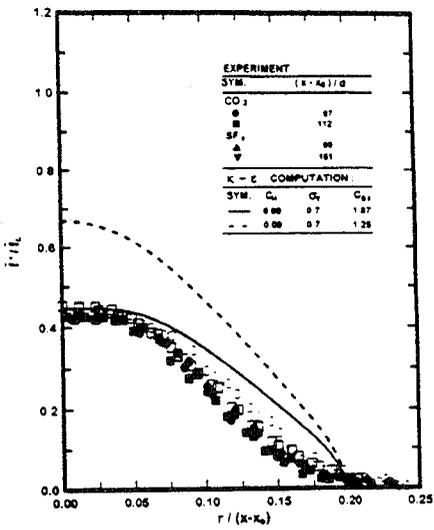


Fig 5. Measured and predicted mixture fraction fluctuations in the self-preserving region.

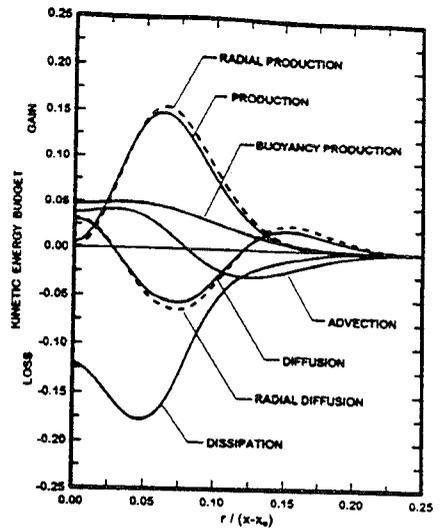


Fig 6. Turbulence kinetic energy budget in the self-preserving region.

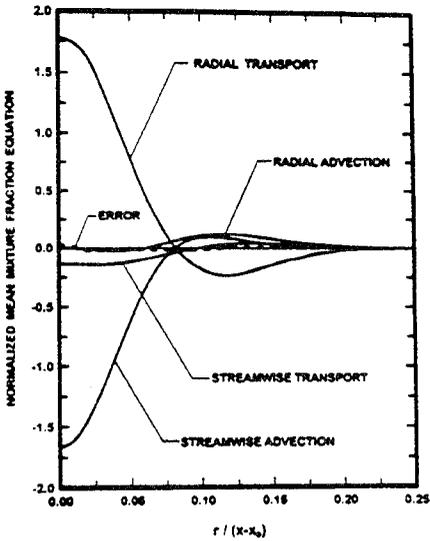


Fig 7. Mixture fraction fluctuation budget in the self-preserving region.

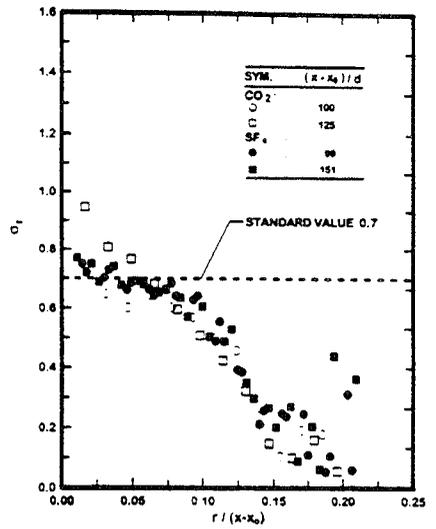


Fig 8. Measured and prescribed turbulence Prandtl/Schmidt number in the self-preserving region.