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Effect of Elastic Modulus of Capping Material on Measured Strength of High-Strength Concrete Cylinders

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ABSTRACT: Studies have shown that end conditions of concrete cylinders tested in compression can have a significant effect on the measured strength of the cylinders, especially when high-strength concrete is used. The ASTM standard for bonded caps has requirements for minimum cube strength of the capping material and maximum cap thickness. However, a study by researchers at the National Ready Mixed Concrete Association (NRMCA) showed that the 50 mm cube strength may not be very useful in determining whether the capping material will perform adequately when testing high-strength concrete. In the study reported in this paper, the dynamic modulus of elasticity and modified cube strength (ASTM C 116) of various capping materials were evaluated as a function of age. The results showed that each capping material has a unique relationship between dynamic elastic modulus and cube strength. The elastic modulus of different capping materials can vary greatly at a given cube strength. For example, at a modified cube strength of 80 MPa, the elastic modulus of neat cement paste, at 30 GPa, was twice the elastic modulus of one sulfur mortar, at only 15 GPa. The elastic modulus of the capping materials was correlated with previously reported cylinder strengths. In cases where the cylinder strength was affected by the capping material, there is evidence that the cylinder strength was related to the modulus of elasticity and not to the cube strength of the capping material.

KEYWORDS: cap thickness, capping materials, compressive strength, elastic modulus, high-strength concrete, modified cube strength, resonant frequency testing

The increased use of high-strength concrete has led to a realization that more care is needed in applying standard testing procedures, developed for normal-strength concrete, to high-strength concrete. When testing high-strength concrete, small differences in testing variables, even within permitted tolerances, can result in low and erratic measured strengths. In past studies, the end conditions have been shown to have significant effects on measured strength (Werner 1958; Lobo et al. 1994; Carino et al. 1994). Current ASTM standards for compressive strength testing of concrete cylinders require that the ends of the test cylinders be plane to within 0.05 mm. Otherwise, the cylinders need to be capped according to the ASTM Practice for Capping Cylindrical Concrete Specimens (C 617-94), or they may be sawed or ground to meet that tolerance. Alternatively, unbonded caps that meet the requirements of the ASTM Practice for Use of Unbonded Caps in Determination of Compressive Strength of Hardened Concrete Cylinders (C 1231-93) can be used for compressive strengths less than 48 MPa. ASTM C 617 has requirements for maximum cap thickness and minimum compressive strength of 50 mm cube specimens of the capping material. For all concrete cylinder compressive strengths, the strength of the capping material must be at least equal to the cylinder strength and no less than 35 MPa. For concrete cylinder strengths of 3.5 to 50 MPa, the maximum average thickness of the caps is 6 mm, and the maximum thickness in any part of the cap is 8 mm. For cylinders stronger than 50 MPa, the maximum average cap thickness is 3 mm, and the maximum thickness in any part of the cap is 5 mm. However, a recent study showed that capping materials with 50 mm cube strengths lower than the concrete strength can be successfully used with no significant reduction or increased variation in the measured cylinder strengths (Lobo et al. 1994). In contrast, some cylinders capped with materials having a higher cube strength than the concrete cylinder strength resulted in lower and more erratic cylinder strengths. Those cylinders had relatively thick caps. It is postulated that another mechanical property, namely the modulus of elasticity, may be more indicative of the performance of capping materials than cube strength.

The objective of the study presented here is to examine the relationship between strength and elastic modulus of capping materials and to explore whether elastic modulus is a better predictor of the performance of a capping material than strength. The discussion begins with a review of the mechanics associated with testing capped cylinders. This is followed by a review of the study by Lobo et al. (1994). Finally, the results of the current study are summarized.

Background

Caps are used in compressive strength tests of concrete cylinders to obtain flat loading surfaces so that the load is transferred evenly onto the cylinder. If the capping material is not stiff enough, the cap cannot effectively distribute the load onto the cylinder. In addition, a low elastic modulus tends to result in high lateral strain in the cap (due to the Poisson's ratio) which may introduce lateral tensile stresses in the ends of the cylinder. Ideally, the elastic modulus of the capping material should be similar to that of the concrete. Even if the strength of the capping material satisfies the requirements of ASTM C 617, the elastic modulus may still be too low for effective load transfer to the cylinder, especially at higher loads.

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Ideally, the bearing blocks of the testing machine should transfer the load uniformly to the cylinder, and this is assumed to be the case when the ends of the cylinders are flat. However, studies have shown that the distribution of stresses within the cylinder is neither uniaxial nor uniform, even when the ends of the cylinder are flat (Troxell 1941; Timoshenko 1970; Ottosen 1984; Carino et al. 1994). In the limiting case of a rigid circular indentor (representing the concrete cylinder) in contact with an elastic half-space (the bearing block), the contact stress is not uniform. Its intensity is given by the following equation developed by Boussinesq in 1885 (Timoshenko 1970, p. 408):

$$\sigma_z = \frac{P}{2\pi a\sqrt{a^2 - r^2}} \qquad (1)$$

where

σ_z = normal stress on contact surface,
P = total load on indentor,
a = radius of indentor, and
r = distance from center of indentor along contact surface.

According to Eq 1, the contact normal stress varies from 50% of the average stress (load divided by the cross-sectional area) at the center of the indentor to an infinite value at the circumference. Since a concrete cylinder is not an absolutely rigid indentor and the bearing block is not a semi-infinite solid, finite element analysis has been used to analyze the stress distribution in a loaded cylinder more accurately (Ottosen 1984; Carino et al. 1994). Similar to the results from the theory of elasticity, the contact stresses are found to be much higher near the circumference of the concrete cylinder in contact with the bearing block. Carino et al. (1994) reported that, at the center of the cylinder, the contact stress is approximately 90% of the average stress and increases nonlinearly towards the perimeter to approximately 140% of the average stress. The linear-elastic finite element analysis was for a 150 × 300 mm concrete cylinder loaded through 200-mm diameter by 100-mm thick steel blocks. It was assumed that there was no sliding between the concrete and steel blocks. Figure 1 shows the stress distributions of both the Boussinesq equation, Eq 1, and the finite element analysis by Carino et al. (1994). The contact stress is given in terms of the ratio of the stress to the average stress, that is, the load divided by the cross-sectional area of the cylinder. The difference between the two stress distributions is because the cylinder is not rigid, as is assumed by the Boussinesq solution. Thus it must be understood that a cap is not intended to produce a uniform stress distribution on the end of the cylinder but rather to produce a distribution that does not differ appreciably from what would be obtained in a cylinder with flat ends.

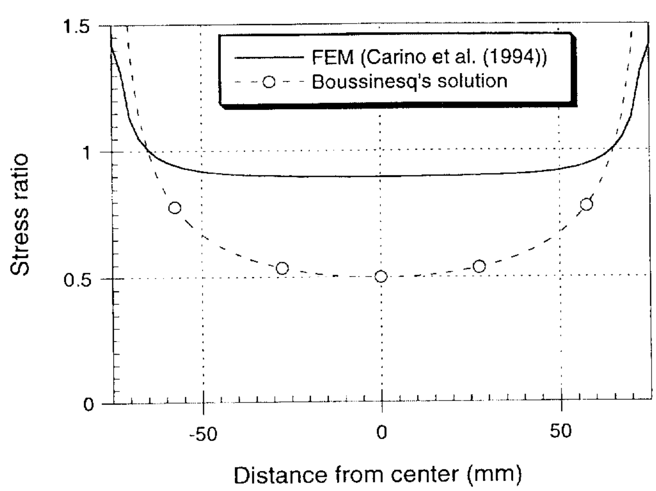

FIG. 1—*Distribution of normal stress on end of a 150 mm by 300 mm cylinder loaded in compression.*

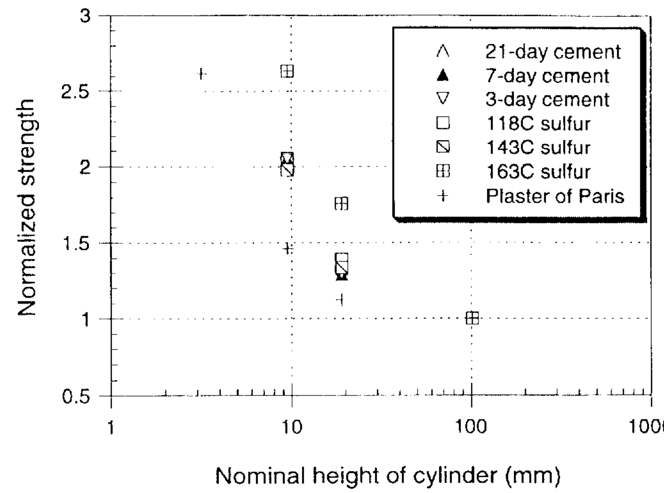

FIG. 2—*Normalized compressive strength of capping material as a function of height of 50-mm-diameter cylinders (adapted from Gaynor and Wedding 1964).*

Another common misconception is that a properly capped cylinder is under uniaxial compression. As the cylinder is loaded in compression, its diameter increases due to Poisson's effect. However, friction between the steel bearing blocks and the ends of the cylinder restrains this expansion, introducing lateral compression to the ends of the cylinder. This restraint and the applied axial load place the ends of the cylinder under triaxial compression. If capping material is present between the bearing block and concrete, the capping material is also under a triaxial stress state. Under triaxial compression, materials can withstand greater stress than under uniaxial compression. This was demonstrated by Gaynor and Wedding (1964), who measured the compressive strength of capping materials as a function of thickness. Cylinders with 50 mm diameters and heights of 3.2, 9.5, 19, and 100 mm were molded from different capping materials. Materials tested included sulfur mortar, neat cement paste, and plaster of Paris. The researchers found that as the height of the cylinder decreased, there was a significant increase in the cylinder strength. Figure 2 shows the cylinder strengths (normalized by the strength of the 100-mm tall cylinders) as a function of height. The variations in normalized cylinder strengths were in agreement with the exception of two materials: the plaster of Paris and the sulfur mortar heated to 163°C. The sulfur mortar that was heated to 163°C had a relatively low strength for the 100-mm-long cylinder, and this accounts for the higher strength ratios for the other cylinder heights. With the exception of the plaster of Paris, the 3.2-mm-thick specimens did not reach any well-defined maximum load, and testing was stopped at stresses between 65 and 132 MPa. A similar study was reported a few years earlier by Helms (Werner 1958). Similar results were found. Helms concluded that 3.2-mm-thick caps made with either sulfur mortar or plaster of Paris will be satisfactory for testing high-strength concrete.

ASTM C 617 currently uses the compressive strength of 50 mm cubes to judge the acceptability of a capping material. However, the stress distribution within 50-mm cubes tested in compression is neither uniaxial nor uniform. The zones affected by the lateral restraint stresses at the top and bottom faces of the cubes overlap. During testing, most of the volume of the cube is in triaxial compression and this accounts for the normally observed "hour-glass" shape of tested cubes. Since the state of stress in the cube tested in compression differs from that in a thin cap, the cube strength may not be an accurate predictor of the performance of the material when used as a cap. It is believed that the main function of the cube test is to serve as a simple means for quality control testing.

Review of NRMCA Study

A study by Lobo et al. (1994) at the National Ready Mixed Concrete Association (NRMCA) focused on the relationships between the end conditions of 100 mm by 200 mm concrete cylinders and the measured cylinder strength. The strengths of the cylinders with ground ends were used as reference strengths. Three nominal concrete strengths were studied: 50, 75, and 120 MPa. Three capping materials were used: neat cement paste and two types of sulfur mortar, designated as SM1 and SM2. SM1 was ordinary, commercially available sulfur mortar, typically used in testing normal-strength concrete. SM2, also commercially available, was advertised as a high-strength sulfur mortar for use with high-strength concrete. Each type of capping material was used with caps of two different nominal thicknesses: 2 mm and 5 mm. Lobo et al. concluded that neat cement paste caps, applied one week prior to testing, resulted in cylinder strengths comparable to ground end conditions for all strength levels and both cap thicknesses. The neat cement paste had a 50 mm cube strength at 7 days of 75 MPa. Note that although the neat cement paste caps performed satisfactorily for all three concrete strength levels and both cap thicknesses, the 50 mm cube strength of 75 MPa did not satisfy the strength requirement given in ASTM C 617 for the 120 MPa concrete.

The sulfur caps for the 50 MPa concrete cylinders were applied two hours before testing. The 50 mm cube strengths of SM1 and SM2 at two hours were 46.9 and 62.7 MPa, respectively. The cylinders capped with thin sulfur mortar caps resulted in strengths 2 to 3% below the average strength of ground cylinders. In contrast, the cylinders with thick caps resulted in strengths up to 7% below the average strength of ground cylinders. Note that SM2 had satisfied the strength requirement of ASTM C 617, but the cylinders with thick caps tested well below the strength of ground cylinders.

The sulfur caps for the 75 and 120 MPa concrete cylinders were applied 7 days prior to testing. The cube strengths of SM1 and SM2 at 7 days were 81.4 and 91.0 MPa, respectively. Both cap thicknesses performed satisfactorily for the 75 MPa concrete capped with the two sulfur mortars; the strengths were within ± 1% of the average strength of ground cylinders. For the 120 MPa concrete, the cylinders with thin caps gave satisfactory results. However, the cylinders with thick sulfur mortar caps resulted in strengths up to 4% below the average of the ground cylinders. There were no significant differences in cylinder strengths for the two types of sulfur mortar. Again, note that the sulfur mortars did not meet the strength requirements of ASTM C 617, but they performed well when thin caps were used.

The results of the study by Lobo et al. 1994 were analyzed further by the authors using analysis of variance (ANOVA) (Mendenhall and Sincich 1992). Each strength level was analyzed separately. The individual cylinder strengths were grouped by end condition (Fig. 3), and ANOVA was used to determine whether there were statistically significant differences among the average strengths of the seven groups. If the ANOVA indicated that differences existed, the Scheffé method for multiple comparisons was used to identify the significant differences (Velleman 1997).

For the 50 MPa concrete cylinders, the following differences were found to be statistically significant at the 95% confidence level, based on the Scheffé test:

- Cylinders with thick caps of SM1 (aged 2 h) were found to be weaker than those with ground ends and those capped with neat cement paste (for both cap thicknesses).
- Cylinders with thick caps of SM2 (aged 2 h) were found to be weaker than those with thick cement paste caps.

For the 75 MPa concrete cylinders, no statistically significant differences where found at the 95% confidence level. Visual inspection of the averages in Fig. 3*b* also shows no practical differences.

For the 120 MPa concrete cylinders, the following statistically significant difference was found:

- Cylinders with thick caps of SM1 were found to be weaker than those with thin caps of SM2.

Although the average strength for cylinders with thick SM1 caps seemed lower than the average strength of the ground cylinders and cylinders with neat cement paste caps, the scatter in all the groups, as shown in Fig. 3*c*, made these differences statistically insignificant.

As mentioned, ASTM C 617 requires that the cube strength of capping materials be equal to or greater than the concrete cylinder strength for concrete strengths greater than 50 MPa. This implies that the performance of a capping material is related to its strength, and that a higher strength capping material would result in better performance. To examine whether this is true, the average cylinder strength was correlated with the capping material strength, as shown in Fig. 4. Average cylinder strengths for the various end conditions were normalized by the average strengths of the ground cylinders. Only two significant correlations were observed. The strength ratios of 50 MPa concrete cylinders with thick caps were correlated to the cube strength of the capping materials. Note that the 50 mm cube strengths of the two sulfur mortars are quite low because the caps were applied only 2 hours before testing. However, the cube strength of SM2 still satisfied the strength requirement of ASTM C 617, i.e., the cube strength was greater than the cylinder strength. For the 75 MPa concrete cylinders, there was a negative correlation between the strength of the cylinders with thin caps and the strength of the capping material. However, this correlation may be of no practical significance because the change in the measured concrete strength was less than 2% as the capping material cube strength varied between 80 and 92 MPa. These observations justify the conclusion by Lobo et al. (1994) that the cube strength by itself is not sufficient as a performance indicator of capping materials and that other mechanical properties should be explored.

Elastic Modulus of Capping Materials

An experimental program was carried out to evaluate further the elastic modulus and the strength of capping materials as a

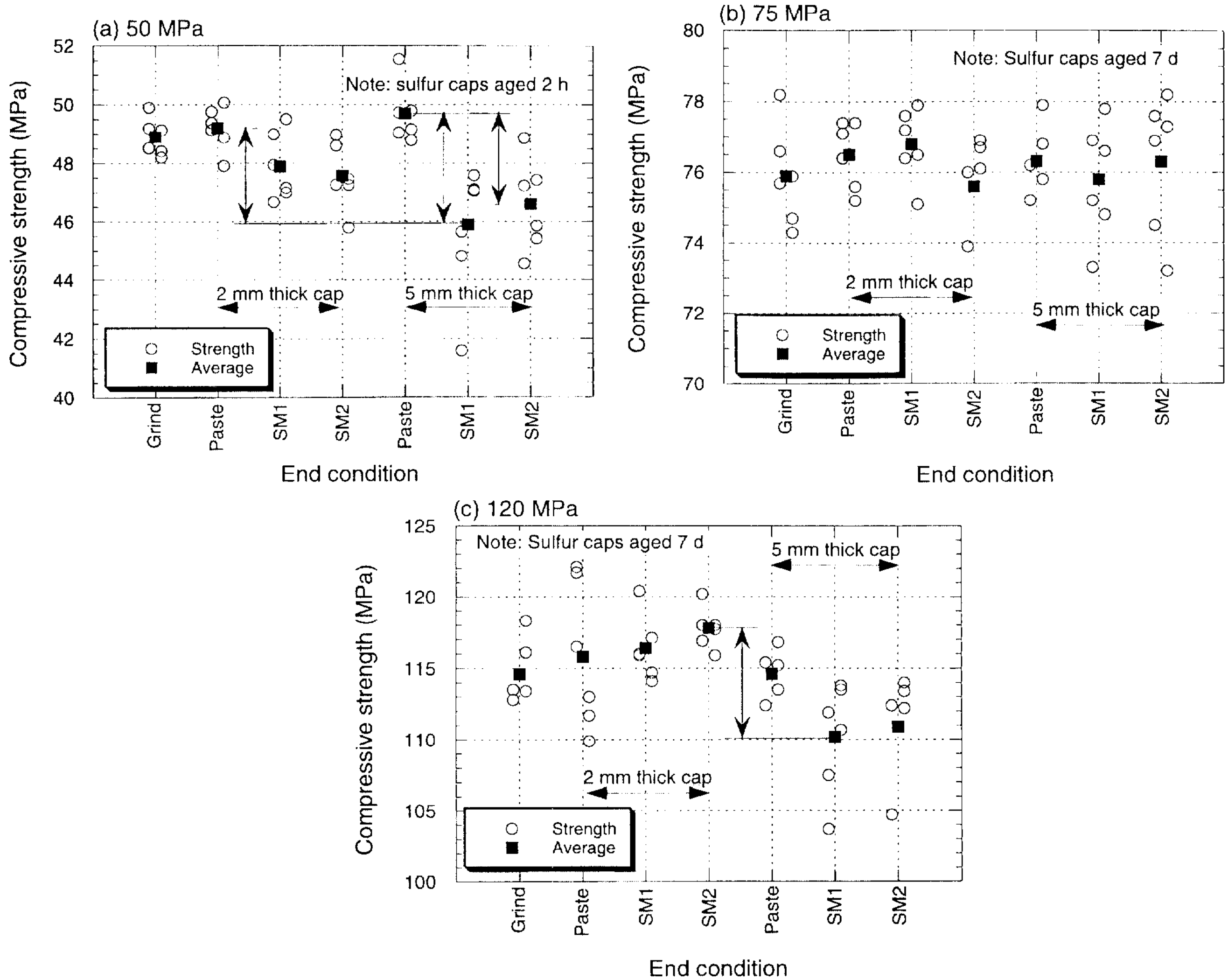

FIG. 3—*Compressive strengths of concrete cylinders with different end condition; statistically significant differences indicated with vertical arrows (data from Lobo et al. 1994).*

function of age. The dynamic elastic modulus was measured using resonant frequency testing of prismatic beams. Capping materials included portland cement paste, high-strength gypsum cement paste, and sulfur mortars. These materials included the sulfur mortars and portland cement used in the NRMCA study (Lobo et al. 1994).

Resonant frequency testing was chosen to measure the elastic modulus of the capping materials because the method is both nondestructive and simple. A nondestructive test allows data to be collected at different ages on the same specimen. In this study, beams of the capping materials were prepared using 285 by 25 by 25 mm molds. The longitudinal resonant frequency was measured according to ASTM C 215 (Test Method for Fundamental Transverse, Longitudinal, and Torsional Frequencies of Concrete Specimens). Resonant frequency testing yields the initial tangent modulus of the material because the frequencies are measured under very small strains (Philleo 1955). The longitudinal frequency is related to the elastic modulus through the compressional wave speed. For the longitudinal mode,

$$C_p = \sqrt{\frac{E_{dyn}}{\rho}} \tag{2}$$

and

$$C_p = 2fL \tag{3}$$

where

C_p = compressional wave speed,
E_{dyn} = dynamic modulus of elasticity,
ρ = density of material,
f = measured longitudinal resonant frequency according to ASTM Test Method for Fundamental Transverse, Longitudinal and Torsional Frequencies of Concrete Specimens (C 215-91), and
L = length of specimen.

From these two equations, the following relationship is obtained:

$$E_{dyn} = 4f^2L^2\rho \tag{4}$$

The lengths of the beams were measured with calipers. The densities of the beams were measured by the water displacement method, with the exception of the high-strength gypsum cement beams.

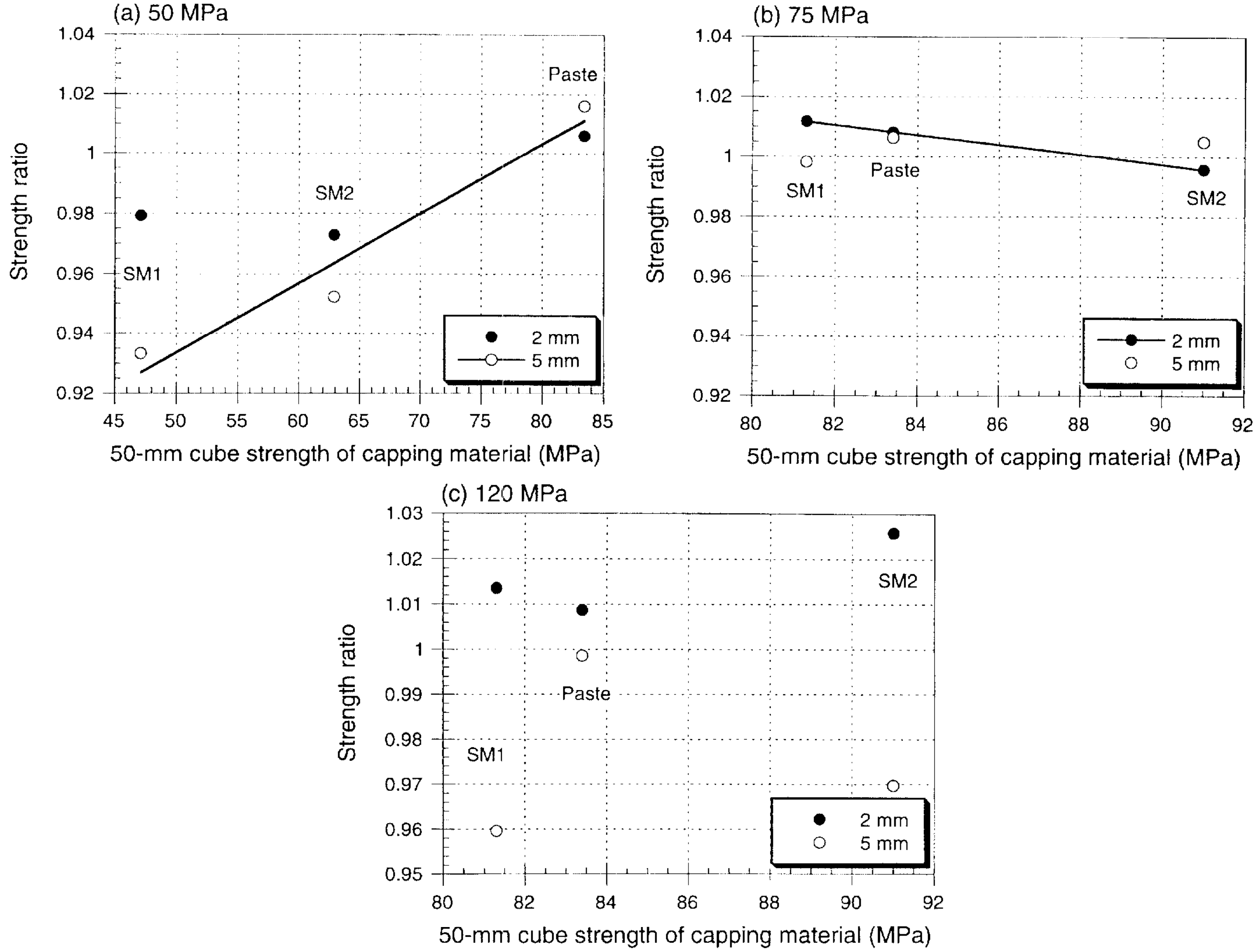

FIG. 4—*Normalized concrete strength vs. 50 mm cube strength of capping material; statistically significant correlations indicated with straight lines (data from Lobo et al. 1994).*

The dimensions of the high-strength gypsum cement beams were measured with calipers, the beams were weighed, and the density calculated.

ASTM C 617 calls for 50 mm cube strength to characterize the strength of the capping material. However, in this study, the procedure in the ASTM Test Method for Compressive Strength of Concrete Using Portions of Beams Broken in Flexure (C 116-90) was used to obtain the *modified* cube strength. Since the stress conditions in cylinder caps and in cubes are different, cube strength is merely an index for comparison. The modified cube strength test was chosen because companion beams cast at the same time in similar molds as those being used for resonant frequency testing would likely have similar properties. If 50 mm cubes were used, the elastic properties of these cubes might not be similar to those of the beams used for resonant frequency testing. Loading apparatus developed at NRMCA, as shown in Fig. 5, was used to apply the compressive load to the beams. The load plates were 25 mm × 25 mm. The experience of the NRMCA staff indicated that the 25 mm modified cube strengths tend to be lower than the 50 mm cube strengths of the same capping material.

The capping materials used in this study were supplied by the NRMCA laboratory to match those used by Lobo et al. (1994).

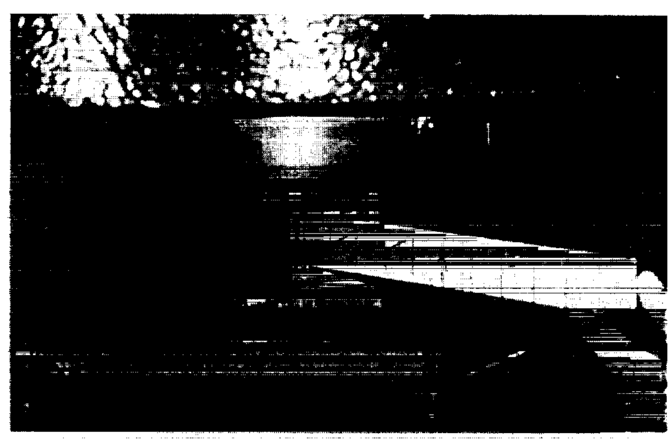

FIG. 5—*Modified cube strength test of prismatic specimen of capping material.*

Portland cement pastes were prepared at two water-cement (*W/C*) ratios: 0.25 and 0.32. The ratio of 0.32 was used to match the cement paste used by Lobo et al. The paste was mixed periodically for approximately 3 h before casting the beam specimens. This delay period is recommended in ASTM C 617 to reduce bleeding and shrinkage of the paste caps. The cement paste beams were cured in lime water at room temperature and removed only when tested.

In addition to the two types of sulfur mortar used by Lobo et al., a third type (SM3) was also tested. SM3, another commercially available product, is commonly used for testing normal-strength concrete. Similar steel molds as those used for preparing the cement paste beams were used to produce the sulfur mortar beams. In this case, an additional steel plate was added to cover the top of the mold and one end of the mold was removed. The beams were cast in a vertical position by pouring the sulfur compound into the open end of the molds. The molten compound was poured in two layers. As the mortar cooled and experienced shrinkage, additional compound was added to the central core that was still in a liquid state. After the beams had cooled to room temperature, the molds were removed. The ends of the beams where the mortar had been poured were ground flat before testing. The beams were left in the laboratory at room temperature throughout the duration of the tests. Note that the elastic properties obtained from sulfur mortar beams should be viewed with caution. The microstructure of the caps and the beams made of sulfur mortar are probably different because of the different cooling rates. The caps cool quickly, and because they are thin, the cooling rate is probably similar throughout most of the cap. The more massive beams cool at a slower rate; in addition, the middle of the beams cool at a slower rate than the surfaces.

A high-strength gypsum cement paste was also tested. This material was not used by Lobo et al. 1994, but was included in this study to examine its elastic modulus and strength gain characteristics. The paste was hand mixed at a water/gypsum (*W/G*) ratio of 0.32. It was not remixed like the portland cement paste because of its relatively fast setting time. The gypsum cement beams were also left in the laboratory with no special curing conditions. The beams were weighed each time they were tested for elastic modulus to account for changes in the water content.

At each test age, one modified cube strength was measured on one beam and the resonant frequency was measured on the companion beam. Test ages for modified cube strength ranged from approximately one hour to seven or 14 days. The modified cube

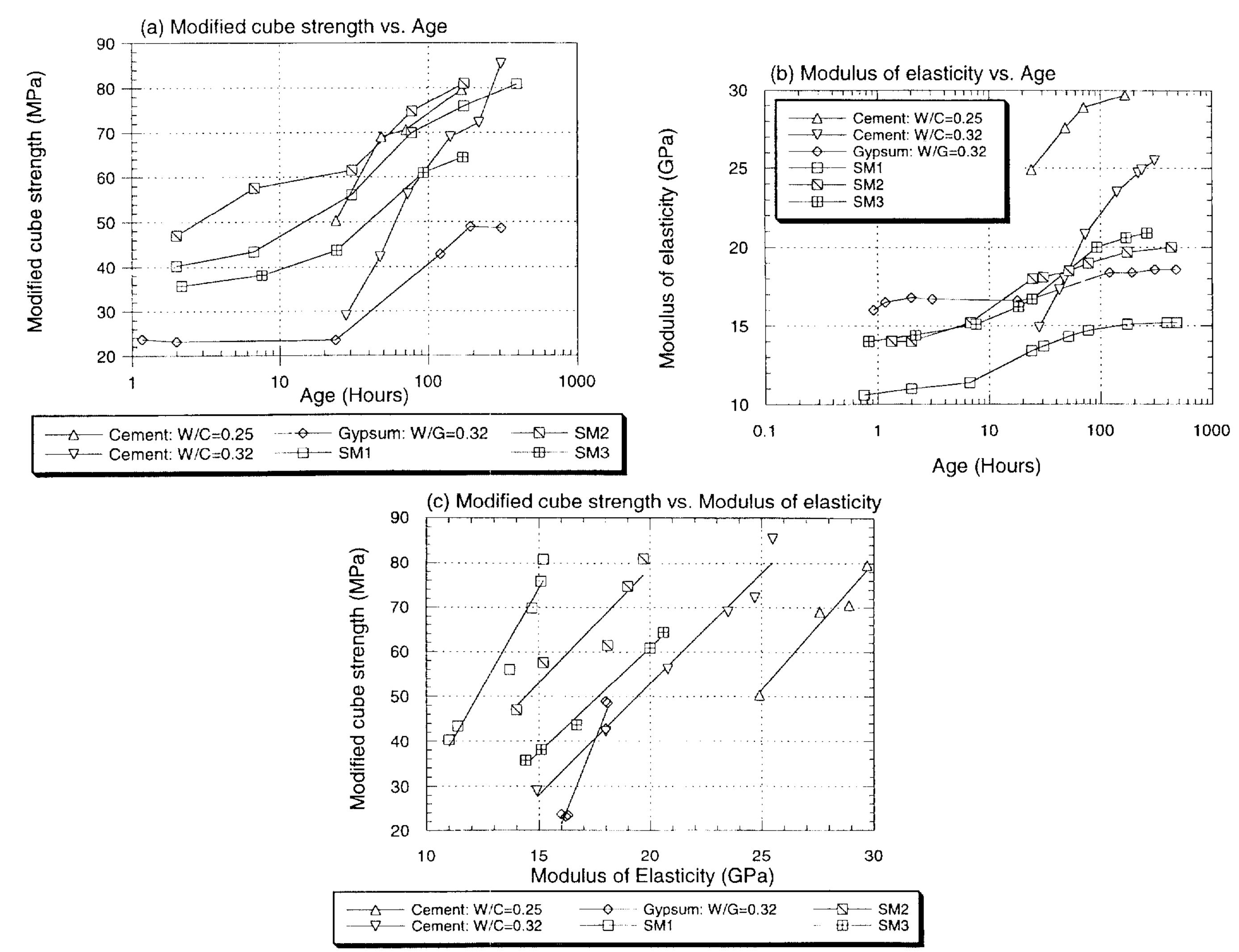

FIG. 6—*Modified cube strength and dynamic elastic modulus of capping materials.*

strengths in this study were about 75% to about 90% of the 50 mm cube strengths reported by Lobo et al. 1994 for the same materials at comparable test ages. In general, 5 to 6 companion values of modified cube strength and elastic modulus were obtained for each material. Summaries of the results as a function of age are shown in Figs. 6*a* and 6*b*. It is seen that there are wide differences in the development of the strength and elastic modulus of these materials. Noteworthy is the large strength gain at later ages in the sulfur mortars. The elastic moduli of the cement pastes were significantly higher than the other materials although the strengths were not dramatically different. Since the modified cube strength and modulus of elasticity were measured at the same age on companion beams, the paired data are plotted in Fig. 6*c*. This plot shows that there is a unique correlation between the elastic modulus and the strength for each material. All the correlations were statistically significant at the 95% confidence level. Note that for a given strength, the elastic modulus varies greatly among the different capping materials. For example, at approximately the same strength, the elastic modulus of the cement paste (W/C = 0.25) is twice the elastic modulus of SM1.

Correlating Elastic Modulus of Capping Materials with Cylinder Strengths

As mentioned earlier, the data of Lobo et al. 1994 revealed no consistent correlations between the measured cylinder strength and cube strength of the capping material (Fig. 4). The present study examines whether there are consistent relationships between cylinder strength and the elastic modulus of the capping material. The elastic moduli of the capping materials measured in this study are correlated to the cylinder strengths from the study by Lobo et al. 1994. Figure 7 shows the normalized cylinder strength versus the elastic modulus of the capping materials. The average cylinder strengths for the different end conditions are normalized with respect to the average strength of the ground cylinders.

For all three strength levels, trends from Fig. 7 show that there is positive correlation between the cylinder strength and modulus of elasticity of the capping materials when 5 mm caps were used. This is in contrast to the previously discussed correlations between the cylinder strength and the 50 mm cube strength of the capping material shown in Fig. 4. For the 50 MPa concrete and 5 mm caps, there is correlation between the normalized cylinder strength and

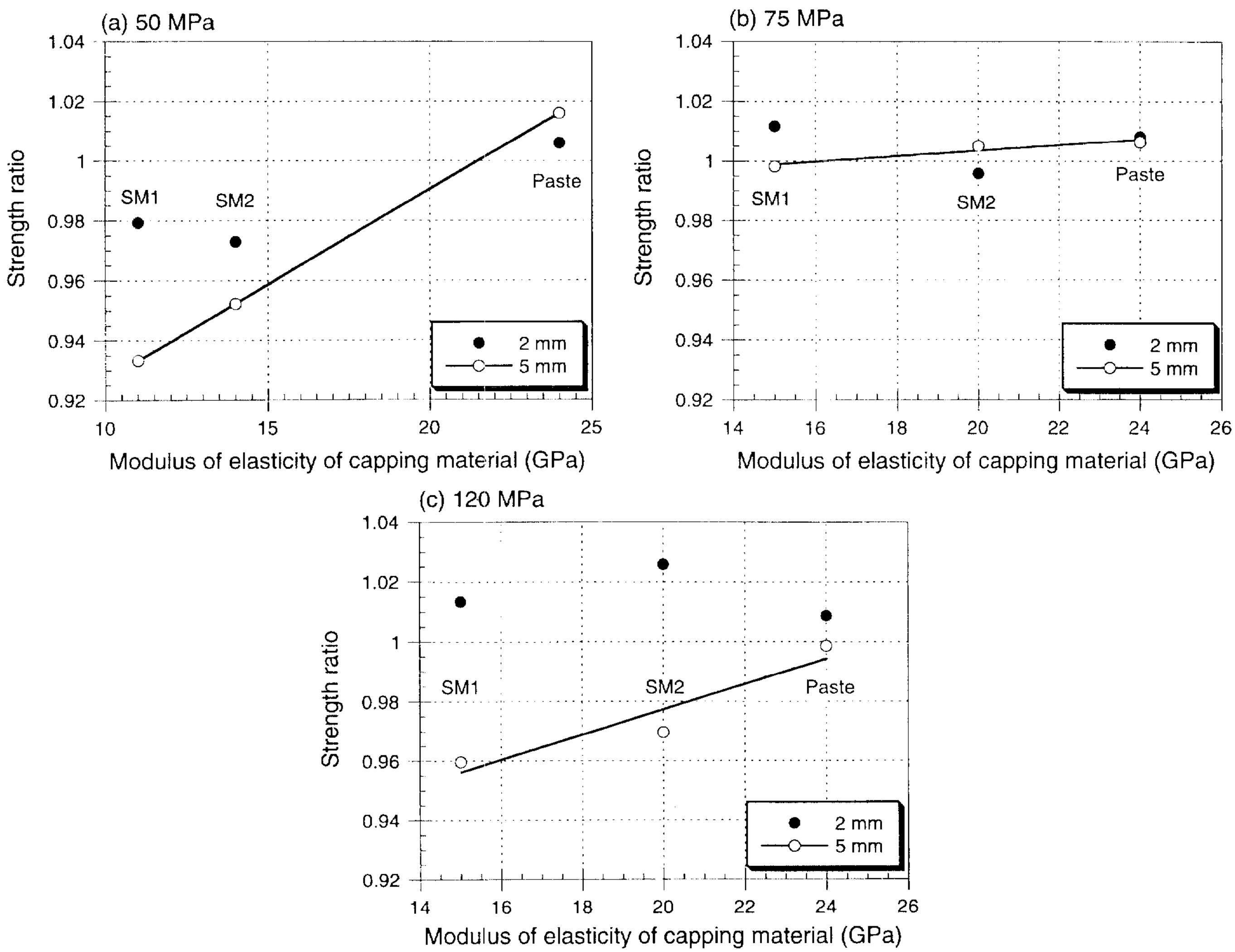

FIG. 7—*Normalized concrete strength vs. dynamic modulus of elasticity of capping material; strength normalized in terms of strength of cylinders with ground ends (strength data from Lobo et al. 1994).*

the cube strength of the capping materials (Fig. 4*a*). A similar correlation can also be the seen between the normalized cylinder strength and the elastic modulus of the capping material (Fig. 7*a*). Since the strength and elastic modulus of the capping materials are correlated (Fig. 6*c*), it is not possible to conclude which of these is the fundamental correlation. For the 75 MPa concrete, there is a negative correlation between the normalized cylinder strength and the strength of the capping materials (Fig. 4*b*). However, Fig. 7*b* shows that there is a positive correlation between the elastic modulus of the capping material and the normalized cylinder strength. Note that although there are correlations in both cases, they may not be of practical significance because of the small differences in normalized strengths for the different end conditions. For the 120 MPa concrete, there is no correlation between the normalized cylinder strength and the cube strength of the capping material (Fig. 4*c*). However, Fig. 7*c* shows that there is a correlation between the normalized cylinder strength and the elastic modulus of the capping material.

This study has shown that at concrete strength levels from 50 to 120 MPa, the cylinders with thick caps resulted in strengths that were correlated positively with the elastic modulus of the capping material. On the other hand, there were no consistent correlations between the cylinder strength and cube strength of the capping materials. This leads to the conclusion that the elastic modulus of the capping materials may be a better performance indicator than the cube strength. Assuming the elastic modulus of the concrete to be 30 to 40 GPa, the limited data presented seem to indicate that $E_{cap} \geq 0.5\ E_{concrete}$ for bonded caps to be effective. Note, however, that for thin caps, neither strength nor elastic modulus had significant effects on the measured cylinder strengths. This reinforces the importance of controlling the smoothness of the cylinder surfaces so that thin caps can be obtained.

Since the present study appears to indicate that the elastic modulus is more important than the strength of the capping material, ASTM C 617 may need to be revised. However, additional studies are needed to verify the conclusion of the present study and to develop appropriate criteria and test methods for elastic modulus of capping materials.

Conclusions

1. In cases where the cylinder strength of concrete is affected by the capping material, there is evidence that the cylinder strength is related to the modulus of elasticity and not to the cube strength of the capping material.
2. Each capping material has a unique relationship between the dynamic elastic modulus and the cube strength. For a given cube strength, the elastic modulus of different capping materials can vary greatly. For example, at a modified cube strength of 80 MPa, the elastic modulus of the low water-cement ratio paste was nearly 30 GPa. In contrast, at the same modified cube strength, the elastic modulus of sulfur mortar, SM1, was only 15 GPa, half that of the cement paste.
3. The use of prismatic beams is convenient for making strength and resonant frequency measurements. Since the cube strength is merely an index of comparison, the modified cube strength test measured on beams should be sufficient as a strength index.

References

Carino, N. J., Guthrie, W. F., and Lagergren, E. S., 1994, "Effects of Testing Variables on the Measured Compressive Strength of High-Strength (90-MPa) Concrete," National Institute of Standards and Technology (U.S.), NISTIR 5405, 139 pp.

Gaynor, R. D. and Wedding, P. A., 1964, "Studies of Capping Procedures for Concrete Cylinders: Effects of Type of Material and Cap Thicknesses (Series J-128)," *Joint Technical Information Letter*, National Sand and Gravel Association No. 227 and National Ready Mixed Concrete Association No. 216, 27 Nov.

Lobo, C. L., Mullings, G. M., and Gaynor, R. D., 1994, "Effects of Capping Materials and Procedures on the Compressive Strength of 100 × 200 mm (4 × 8 in.) High Strength Concrete Cylinders," *Cement, Concrete, and Aggregates*, Vol. 16, No. 2, Dec., pp. 173–180.

Mendenhall, W. and Sincich, T., 1992, *Statistics for Engineering and the Sciences*, 3rd ed., Dellen Publishing Co., San Francisco, Chapter 14.

Ottosen, N. S., 1984, "Evaluation of Concrete Cylinder Tests Using Finite Elements," *ASCE Journal of Engineering Mechanics*, Vol. 110, No. EM-3, March, pp. 465–481.

Philleo, R. E., 1955, "Comparison of Results of Three Methods of Determining Young's Modulus of Elasticity of Concrete," *ACI Journal*, Vol. 26, No. 5, Jan., pp. 461–469.

Troxell, G. E., 1941, "The Effects of Capping Methods and End Conditions Before Capping upon Compressive Strength of Concrete Cylinders," *ASTM Proceedings*, Vol. 41, pp. 1039–1052 (includes discussions by Vidal, Murray, and Durelli).

Velleman, P. F., 1997, *Data Desk® User Manual*, Vol. 03, Statistics Guide, Data Description Inc., Ithaca, NY.

Werner, G., 1958, "The Effects of Capping Material on the Compressive Strength of Concrete Cylinders," *ASTM Proceedings*, Vol. 58, pp. 1166–1186 (includes discussion by Helms).