Pergamon

Cement and Concrete Research, Vol. 27, No. 4, pp. 495–500, 1997

0008-8846/97 $17.00 + .00

PII S0008-8846(97)00029-X

STRESS, DISPLACEMENT, AND EXPANSIVE CRACKING AROUND A SINGLE SPHERICAL AGGREGATE UNDER DIFFERENT EXPANSIVE CONDITIONS

E.J. Garboczi
National Institute of Standards and Technology
Building Materials Division, 226/B350
Gaithersburg, Maryland 20899 USA

(Communicated by J.P. Skalny)
(Received January 15, 1997; in final form January 23, 1997)

ABSTRACT

There seems to have been some confusion in recent years regarding the stress and displacements induced in a concrete material when either the matrix, the aggregate, or a shell around the aggregate is expansive. The case of a single spherical aggregate, surrounded by a single shell of arbitrary thickness, all of which are embedded in a matrix, can be solved exactly for any combination of elastic moduli and expansive strains. Solutions of this problem under various choices of parameters are used to illustrate the features of stress, displacement, and expansive cracking that are possible. These solutions can then be used to help better understand the features seen under the microscope in concretes that have been damaged by different expansive forces. © *1997 Elsevier Science Ltd*

Introduction

There are many ways in which a concrete can be damaged by chemical action, including various kinds of sulfate-attack, alkali-aggregate reaction, and others [1-3]. Often, the chemical attack causes one or more phases in the concrete material, either in the paste or in the aggregates, to grow physically. This growth induces tensile strains that cause cracking that can eventually lead to severe damage to the material. The kinds of cracking that are produced depend on which constituent has been induced to grow. Often the crack pattern that is seen under the microscope is used to try to diagnose which deleterious mechanism was responsible for the damaged concrete. There seems to have been some confusion in recent years concerning what kinds of cracking imply what kinds of physical mechanisms.

The simple, analytically soluble case of a single isolated spherical aggregate, surrounded by a shell of arbitrary thickness, all embedded in a uniform matrix, where each of the three phases can have arbitrary elastic moduli and expansive strains, can be illustrative of what sort of stresses and expansions one would expect to see given various choices of elastic and expansive parameters. In a real concrete, the close positioning of many aggregate particles will of course play a role, but the isolated aggregate case will dominate the overall qualitative features of the stress and displacements that will be seen in the real material. In this paper, we show the general equations for this case, and illustrate the kinds of stress and displacement patterns that arise from different choices of the elastic moduli and expansive strain parameters. Some of these calculations, without the shell phase, have been done before [4].

Basic Equations

Figure 1 shows a cut through the center of the spherical aggregate, and defines the terminology used. We take s = a + h, where a is the radius of the aggregate, h is the thickness of the layer around the aggregate, and b is the overall radius of the composite system being considered. The layer around the aggregate is not necessarily to be thought of as the interfacial transition zone, but only as a parameter of the problem to be discussed. Figure 1 is not to scale, since in this dilute limit the quantity $c = a^3/b^3$, the volume fraction of the aggregate phase, should be small (<0.01-0.03).

Now suppose that at least one of the phases has a non-zero value of expansive strain, so that displacements and stresses will be set up in the system. In spherical polar coordinates, the radial component of displacement, denoted u, will be the only non-zero displacement and will be a function of r only. The origin is taken at the center of the aggregate. In terms of u, the three diagonal components of the strain tensor (all shear strains are zero) are: $\epsilon_{rr} = \partial u/\partial r$ and $\epsilon_{\theta\theta} = \epsilon_{\phi\phi} = u/r$, where θ and ϕ are the spherical polar coordinate angular variables. In the i'th phase, the two independent ($\sigma_{\theta\theta} = \sigma_{\phi\phi}$) diagonal components of the stress tensor are (all shear stresses are zero):

$$\sigma_{rr} = (K_i + \frac{4}{3}G_i)\,\frac{\partial u}{\partial r} + 2\,(K_i - \frac{2}{3}G_i)\,\frac{u}{r} - 3K_i\,\epsilon_i^o \tag{1}$$

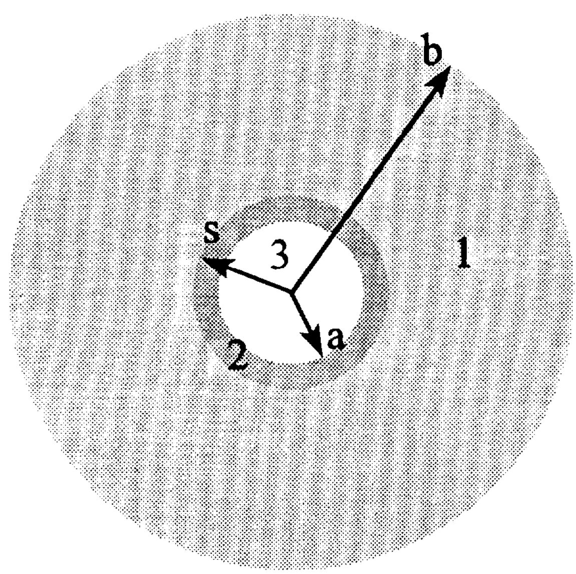

FIG. 1.

Showing the system analyzed in the paper: (1) matrix, s < r < b (2) shell, a < r < s, and (3) aggregate, 0 < r < a, where r is the radial coordinate. The i'th phase has bulk modulus K_i, shear modulus G_i, and expansive strain ϵ_i^o.

$$\sigma_{\theta\theta} = (K_i - \frac{2}{3}G_i)\frac{\partial u}{\partial r} + (2K_i + \frac{2}{3}G_i)\frac{u}{r} - 3K_i\,\epsilon_i^o \tag{2}$$

It has been shown before [5-7] that in phase i the displacement u will have the form $u = \alpha_i r + \beta_i/r^2$, where α_i and β_i are constants, different for each phase. Of course in phase 3, the aggregate, $\beta_3 = 0$, because the displacement must not diverge. So there are five coefficients that must satisfy five equations, in order for there to be a solution to this problem. The five equations come from the fact that the displacement and the radial stress must be continuous at the interfaces ($r = a$ and $r = a + h$), and the radial stress must be zero at the free boundary ($r = b$). These five equations are:

$$\alpha_2 + \frac{\beta_2}{a^3} - \alpha_3 = 0 \tag{3}$$

$$\alpha_1 + \frac{\beta_1}{s^3} - \alpha_2 - \frac{\beta_2}{s^3} = 0 \tag{4}$$

$$K_2\alpha_2 - \frac{4}{3}\frac{G_2\beta_2}{a^3} - K_3\alpha_3 - K_2\epsilon_2^o + K_3\epsilon_3^o = 0 \tag{5}$$

$$K_1\alpha_1 - \frac{4}{3}\frac{G_1\beta_1}{s^3} - K_2\alpha_2 + \frac{4}{3}\frac{G_2\beta_2}{s^3} + K_2\epsilon_2^o - K_1\epsilon_1^o = 0 \tag{6}$$

$$K_1\alpha_1 - \frac{4}{3}\frac{G_1\beta_1}{b^3} - K_1\epsilon_1^o = 0 \tag{7}$$

Equation (3) comes from continuity of u at $r = a$, eq. (4) is from continuity of displacement at $r = a + h$, eq. (5) represents continuity of radial stress at $r = a$, eq. (6) is from continuity of radial stress at $r = a + h$, and eq. (7) comes from the vanishing of radial stress at $r = b$. Taking various choices of the parameters, these equations can easily, though a bit tediously, be solved for the values of α and β in each phase. The stress, strain, and displacement anywhere can then be easily found. For mathematical convenience, the Poisson's ratio of each phase is taken to be 0.2, so that in each phase, $K_i = 4G_i/3$, and $K_1 = K_2 = 1$, and $K_3 = 4$ (arbitrary units). The qualitative aspect of the solutions are not affected by this (physically reasonable) choice. It is easier to solve eqs. (3)-(7) for a given choice of parameters, then to develop a general solution into which different choices of parameters are substituted. The general solution of eqs. (3)-(7) is rather complicated [5].

Results for Various Parameter Choices

Uniform Matrix Expansion. Several degradation mechanisms, including freeze-thaw, some kinds of sulfate attack, and a hypothesized mechanism for so-called delayed ettringite formation [8], involve a uniform, on average, expansion of the matrix. Let us examine what effect this scenario has on stresses and displacements. Take $h = 0$, so that there is effectively no shell phase, and let $\epsilon_1^{\circ} = \epsilon$, and $\epsilon_3^{\circ} = 0$. Combining equations (3)-(7) into three equations for the three variables α_3, α_1, and β_1, we obtain

$$\alpha_1 = \frac{(5 - c)\epsilon}{5 + 3c}, \quad \beta_1 = \frac{-4a^3\epsilon}{5 + 3c}, \quad \alpha_3 = \frac{(1 - c)\epsilon}{5 + 3c} \tag{8}$$

These equations are exact, although, to be consistent with the dilute limit hypothesis, the expressions in eq. (8) should really be expanded to O(c). Using these solutions in equations (1) and (2) shows that σ_{rr} is tensile, and has its greatest value right at the matrix-aggregate interface. If the stress generated is large enough, the aggregate should break away from the matrix along the interface. The tangential stress, $\sigma_{\theta\theta}$, is equal to σ_{rr} in the aggregate, but is always compressive in the matrix, so there should be no radial cracking in the matrix. If the aggregate should break away from the matrix, then there will be a non-zero displacement of the rim of the matrix. This can be computed using the same equations, but now eq. (3) can be ignored, and eq. (5) becomes the vanishing of radial stress at the (now) free boundary at r = a. Solving for the radial displacement at r = a, we find that $u(r = a) = \epsilon a$. Since the outer edge of the detached aggregate will have zero displacement, there will be a gap around the aggregate of width ϵa, which is proportional to the aggregate diameter. In a real concrete with many aggregates packed closely together, as long as each aggregate breaks away from the matrix, each gap will, on the average, have a width proportional to the radius of the aggregate. If some aggregates remain attached to the cement paste matrix, the stress fields will be changed somewhat, but the overall result should still be approximately true.

Thin Expansive Shell around the Aggregate. One proposal for the mechanism for damage during so-called delayed ettringite formation is the formation of expansive ettringite right at the cement paste-aggregate interface [8,9]. The expansion of this thin shell then produces tensile radial stresses at the interface, which causes the aggregates to de-bond with the matrix, so that a rim of free space will be observed around the aggregate. By letting $h/a \ll 1$, so as to simulate a thin shell, and $\epsilon_1^{\circ} = \epsilon_3^{0} = 0$, $\epsilon_2^{0} = \epsilon$, we can investigate the kinds of stresses and displacements that are produced in this scenario. Solving eqs. (3)-(7) for this choice of parameters, we find that, to first order in h/a,

$$\begin{gathered}\alpha_1 = \frac{\epsilon \frac{15}{2}\frac{h}{a}c}{5 + 3c}, \qquad \beta_1 = b^3 \alpha_1, \qquad \alpha_3 = \frac{5\epsilon c \frac{h}{a}}{5 + 3c} \\ \alpha_2 = \frac{1}{2}\epsilon \frac{\left(5 + 3c + 15c\frac{h}{a}\right)}{5 + 3c}, \quad \beta_2 = -\frac{1}{2}\epsilon a^3 \frac{\left(5 + 3c + 9c\frac{h}{a}\right)}{5 + 3c}\end{gathered} \tag{9}$$

The radial stress is tensile, with again a maximum at the aggregate-cement paste matrix interface. Cracking, if any, will again be circumferential. Separating the aggregate from the matrix by taking $\sigma_{rr} = 0$ at r = a, we can find the displacement of the cement paste edge at r = a by re-solving the equations under this assumption. We find that the thickness of the ensuing gap at the aggregate edge will be, to leading order in h/a, $u(r = a) = 3hc\epsilon/(1-c)$, which is proportional to the thickness of the expansive region, not the aggregate radius. The assumption of $h \ll a$ simplified the mathematics, but approximately the same result will be found for larger values of h.

Expansive Aggregate. In an alkali-aggregate reaction, the gel may form inside the aggregate, in aggregate pores or cracks. The expansive forces then act only on the aggregate, not the cement paste matrix. We can model this case by setting $\epsilon_1^o = \epsilon_2^0 = 0$, $\epsilon_3^0 = \epsilon$, and letting h = 0 to remove the layer. Equations (3)-(7) then reduce to three equations for α_1, β_1, and α_3, with the result

$$\alpha_1 = \frac{4c\epsilon}{5+3c}, \quad \beta_1 = \frac{4a^3\epsilon}{5+3c}, \quad \alpha_3 = \frac{4(1+c)\epsilon}{5+3c} \tag{10}$$

The radial stress is found to compressive everywhere. The tangential stress $\sigma_{\theta\theta}$ is compressive in the aggregate but tensile in the matrix, with its maximum tensile value at r = a. Therefore any cracking will be radially outward from the aggregate, and so no open gaps around the aggregates should in general be observed. In actual instances where expansive gel forms inside the aggregate, there will be parts of the aggregate under tensile stress, since the whole aggregate will not uniformly expand, so that there could be cracking within the aggregate as well. Treatment of random expansive centers within the aggregate cannot be done analytically, and must be handled by a numerical method [10,11].

Discussion

When different parts of the microstructure of a concrete undergo expansive growth, different kinds of stress and displacement patterns result. Many scenarios can be studied with the equations developed in this paper, since eqs. (1)-(7) are completely general for linear, isotropic elastic materials. Viscoelastic effects have been ignored in this paper, though they do play a major role in these kinds of deleterious processes [12,13]. In this paper, the shell phase was not thought of as the interfacial transition zone, but it could be, allowing the interplay between interfacial zone cement paste, bulk cement paste, and aggregate to be studied [5].

Under the assumptions of this paper, it is clear that uniform, on average, matrix expansion, followed by circumferential cracking between aggregate and matrix due to radial stress in the matrix, will lead to open gaps surrounding the aggregates, whose width is proportional to the aggregate radius. If this circumferential cracking was caused by the expansion of a thin layer located at the interface, like in some models of delayed ettringite formation-induced damage, then the widths of the gaps observed would be proportional to the layer thickness, and not the aggregate radii. Aggregate expansion leads to radial cracking, and so no gaps will appear at all. All these results are of course subject to some modification due to the many aggregates that appear in a real, non-dilute concrete, but the major qualitative differences between the kinds of cracking produced by expansion of different parts of the microstructure will remain the same as those predicted by this simple dilute model [11]. This makes the results of this paper useful for interpreting cracks produced by unknown deleterious processes in field concretes observed under the microscope. Non-spherical aggregate shapes will also not change the qualitative aspects of the problem studied in this paper.

Acknowledgements

I would like to thank the National Science Foundation Science and Technology Center for Advanced Cement-Based Materials for partial support of this work, and Dr. Jan Skalny for suggesting this problem to me.

References

1. J.R. Clifton and L.I. Knab, NISTIR 89-4086 U.S. Dept. of Commerce, Gaithersburg, MD, 1989.
2. H.F.W. Taylor, Cement Chemistry Academic Press, London, 1990.
3. P.K. Mehta and P.J.M. Monteiro, Concrete: Structure, Properties, and Materials, Prentice-Hall, Englewood Cliffs, New Jersey, 1993.
4. P. Goltermann, Amer. Conc. Inst. Mater. J. 91, 543 (1994).
5. C.M. Neubauer, H.M. Jennings, and E.J. Garboczi, J. of Adv. Cem.-Based Mater. 4, 6 (1996).
6. S.P. Timoshenko and J.N. Goodier, Theory of Elasticity McGraw-Hill, New York, 1970.
7. E. Herve and A. Zaoui, Int. J. Engng. Sci. 31, 1 (1993).
8. J. Skalny, V. Johansen, N. Thaulow, and A. Palomo, "DEF as a form of sulfate attack," Materiales de Construccion 46, No. 242, 1996, in press.
9. V. Johansen, N. Thaulow, and J. Skalny, Adv. Cem. Res. 5, 23 (1993).
10. E.J. Garboczi, "Finite element and finite difference codes for computing the linear electrical and elastic properties of digital images of random materials," National Instuitute of Standards and Technology Internal Report (1996), in press.
11. E.J. Garboczi and P.E. Stutzman, "Simulation of stress, displacement, and expansive cracking in 2-d images of model and real mortars under different expansive conditions," in preparation.
12. C.F. Ferraris, J.R. Clifton, E.J. Garboczi, and F.L. Davis, "Stress due to alkali-silica reactions in mortars," in Mechanisms of Chemical Degradation of Cement-Based Systems, Fall, 1995, Materials Research Society Symposium Proceedings (Elsevier, 1996), in press.
13. C.F. Ferraris, J.R. Clifton, E.J. Garboczi, and F.L. Davis, "Stress due to alkali-silica reactions in mortars: Further results," 4th ASCE Conference on Materials, Washington, D.C., 1996 (in press).