EXTREME WIND DISTRIBUTION TAILS: A "PEAKS OVER THRESHOLD" APPROACH

By E. Simiu[1] and N. A. Heckert[2]

ABSTRACT: We seek to ascertain whether the reverse Weibull distribution is an appropriate extreme wind-speed model by performing statistical analyses based on the "peaks over threshold" approach. We use the de Haan method, which was found in previous studies to perform about as well or better than the Pickands and cumulative mean exceedance methods, and has the advantage of providing estimates of confidence bounds. The data are taken principally from records of the largest daily wind speeds obtained over periods of 15 to 26 years at 44 U.S. weather stations in areas not subjected to mature hurricane winds. From these records we create samples with reduced mutual correlation among the data. In our opinion, the analyses provide persuasive evidence that extreme wind speeds are described predominantly by reverse Weibull distributions, which unlike the Gumbel distribution have a finite upper tail and lead to reasonable estimates of wind load factors. Instructions are provided for accessing the data and attendant programs.

INTRODUCTION

A fundamental theorem in extreme value theory states that sufficiently large values of independent and identically distributed variates are described by one of three extreme value distributions: the Fréchet distribution (with an infinite upper tail), the Gumbel distribution (whose upper tail is also infinite, but shorter than the Fréchet distribution's), and the reverse (negative) Weibull distribution, whose upper tail is finite (Castillo 1988).

The wind loading provisions of the *American National Standard ANSI A58.1*-(1972) were based on the assumption that a Fréchet distribution best fits nontornadic extreme wind speeds blowing from any direction in regions not subjected to mature hurricane winds. However, an extensive study concluded that the Gumbel distribution is a more appropriate model (Simiu et al. 1978). It is a physical fact that extreme winds are bounded, and one would expect the probabilistic model to reflect this fact. Therefore, to the extent that an extreme value distribution would be a reasonable model of extreme wind behavior, one would expect the best-fitting distribution to have a finite tail, that is, to be a reverse Weibull distribution.

In addition to the certainty that wind speeds are bounded, there is at least one other indication, albeit indirect, that the Gumbel model might be an inappropriate model of extreme wind behavior. Estimated safety indices for wind-sensitive structures based on the Gumbel model imply unrealistically high failure probabilities (Ellingwood et al. 1980). This is likely due to (at least in part) the use in those estimates of a distribution with an unrealistically long (infinite) upper tail.

In this paper we seek to ascertain whether the reverse Weibull distribution is an appropriate extreme wind-speed model by performing statistical analyses based on the "peaks over threshold" approach. This approach enables the analyst to use all the data exceeding a sufficiently high threshold, and is more effective than the classical approach, which uses only the largest value in each of a number of basic comparable sets called epochs (typically, for extreme wind analysis an epoch consists of one year). To illustrate this point, consider, for example, two successive years in which the respective largest wind speeds are 30 and 45 m/s. Assume that in the second year winds with speeds of 31, 37, 41, and 44 m/s were also recorded (at dates separated by sufficiently long intervals to view the data as independent). For the threshold theory, for a 30 m/s threshold, the two years would supply six data. The classical theory would make use of only two data points. It may be argued that, by choosing a somewhat lower threshold, the number of data points used in the analysis could be considerably larger than six in our example. However, excessive lowering of the threshold would introduce correlation to the sequence and render the underlying assumptions of asymptotic extreme value theory invalid. Simulations reported by Gross et al. (1994) suggest that, in samples taken from normal or extreme value populations, optimal results are obtained if the threshold is chosen so that the number of exceedances is of the order of 10 per year.

Given a sample of data exceeding a sufficiently high threshold, the analyst using the peaks over threshold approach must choose an appropriate estimation method. In this paper we use the estimation method proposed by de Haan (1994). Our choice is based on two reasons. First, Monte Carlo simulations suggest that the de Haan method performs about as well or better than two available alternative methods, the Pickands method and the cumulative mean exceedance method (Gross et al. 1994). Second, the de Haan method has the advantage of providing estimates of confidence bounds.

Data used in this paper are taken principally from records of the largest daily wind speeds obtained over periods of 15 to 26 years at 44 U.S. weather stations in areas not subjected to mature hurricane winds. A storm system usually affects a given location for longer than one day, so that wind-speed data recorded on two or even more consecutive days are not necessarily independent. We describe the data samples and our procedure for creating, from the samples of largest daily speeds, samples with reduced mutual correlation among the data. In addition to samples of daily data, we describe and analyze 115 samples consisting only of the largest yearly speeds recorded over periods of 18 to 54 years at locations not subjected to mature hurricane winds. To our knowledge no tornado winds have affected any of our data. All the data samples used in our analyses are available in an anonymous data file. Instructions for accessing the file and attendant programs are given in Appendix I.

In our opinion, the results presented in this paper provide persuasive evidence that extreme wind speeds of extratropical origin and excluding tornadoes are described predominantly by reverse Weibull distributions. This result is in itself useful from a structural engineering viewpoint, and we discuss its potential implications for the estimation of load factors for

[1]NIST Fellow, Buil. and Fire Res. Lab., Nat. Inst. of Standards and Technol., Gaithersburg, MD 20899.

[2]Comp. Specialist, Computing and Appl. Math. Lab., Nat. Inst. of Standards and Technol., Gaithersburg, MD.

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wind-sensitive structures. Our analyses also suggest that estimates of extreme wind speeds based on the reverse Weibull model may not be obtainable with sufficient confidence from the information and by the method used in this paper. This suggests the need for: (1) data samples based on longer recording periods; and (2) more efficient estimation methods.

The paper is organized as follows. We first review basic theoretical results pertaining to the peaks over threshold approach. Next we briefly review the de Haan method. We then describe the data used in the analyses, and present the main results of our investigation. These are used for a discussion of the load factors issue. The last section presents our conclusions.

"PEAKS OVER THRESHOLD" APPROACH

Generalized Pareto Distribution

The generalized Pareto distribution (GPD) is an asymptotic distribution developed by using the fact that exceedances of a sufficiently high threshold are rare events to which the Poisson distribution applies. The expression for the GPD is

$$G(y) = \mathrm{Prob}[Y \le y] = 1 - \{[1 + (cy/a)]^{-1/c}\} \quad a > 0, \quad [1 + (cy/a)] > 0 \tag{1}$$

Eq. (1) can be used to represent the conditional cumulative distribution of the excess $Y = X - u$ of the variate X over the threshold u, given $X > u$ for u sufficiently large (Pickands 1975). The cases $c > 0$, $c = 0$, and $c < 0$ correspond (respectively) to Fréchet (type II extreme value), Gumbel (type I extreme value), and reverse Weibull (type III extreme largest values) domains of attraction. For $c = 0$ the expression between braces is understood in a limiting sense as the exponential $\exp(-y/a)$ [Castillo (1988), p. 215].

Given the mean $E(Y)$ and standard deviation $s(Y)$ of the variate Y

$$a = 1/2E(Y)\{1 + [E(Y)/s(Y)]^2\}; \; c = 1/2\{1 - [E(Y)/s(Y)]^2\} \tag{2, 3}$$

(Hosking and Wallis 1987).

Gumbel and Reverse Weibull Distributions

We recall that the expressions for the Gumbel and reverse Weibull distributions are, respectively

$$F_G(x) = \exp\{-\exp[-(x - \mu_G)/\sigma_G]\} \tag{4}$$

$$F_W(x) = \exp\{-[(\mu_W - x)/\sigma_W]^\gamma\}, \; x < \mu_W \tag{5}$$

For the Gumbel distribution, relations between the distribution parameters and the expected value $E(X)$ and standard deviation $s(X)$ are

$$\sigma_G = (6^{1/2}/\pi)s(X); \; \mu_G = E(X) - 0.57722(6^{1/2}/\pi)s(X) \tag{6, 7}$$

For the Weibull distribution

$$\sigma_W = s(X)/\{\Gamma(1 + 2/\gamma) - [\Gamma(1 + 1/\gamma)]^2\}^{1/2}; \; \mu_W = E(X) + \sigma_W\Gamma(1 + 1/\gamma) \tag{8, 9}$$

where Γ = gamma function (Johnson and Kotz 1972). For example, for $E(X) = 50$, $s(X) = 6.25$, and $\gamma = 2$, $\sigma_W = 13.49$ and $\mu_W = 61.96$. The tail-length parameter γ is related to the parameter c in the GPD distributions as follows:

$$\gamma = -1/c \tag{10}$$

(Smith 1989).

Mean recurrence intervals of variate X as functions of GPD parameters and exceedance rate: The mean recurrence interval R of a given wind speed, in years, is defined as the inverse of the probability that the wind speed will be exceeded in any one year [see, e.g., Simiu and Scanlan (1996)]. In this section we give expressions that allow the estimation from the GPD of the value of the variate corresponding to any percentage point $1 - 1/(\lambda R)$, where λ is the mean crossing rate of the threshold u per year (i.e., the average number of data points above the threshold u per year). Set

$$\mathrm{Prob}(Y < y) = 1 - 1/(\lambda R) \tag{11}$$

Using (1)

$$1 - (1 + cy/a)^{-1/c} = 1 - 1/(\lambda R) \tag{12}$$

Therefore

$$y = -a[1 - (\lambda R)^c]/c \tag{13}$$

(Davison and Smith 1990). The value being sought is

$$x_R = y + u \tag{14}$$

where u = threshold used in the estimation of c and a.

DESCRIPTION OF DE HAAN ESTIMATION METHOD

Let the number of data above the threshold be denoted by k, so that the threshold u represents the $(k + 1)$th highest data point(s). We have $\lambda = k/n_{yrs}$, where n_{yrs} denotes the length of the record in years. The highest, second, . . . , kth, $(k + 1)$th highest variates are denoted by $X_{n,n}$, $X_{n-1,n}$, $X_{n-(k+1),n}$, $X_{n-k,n} \equiv u$, respectively. Compute the quantities

$$M_n^{(r)} = \frac{1}{k}\sum_{i=0}^{k-1}[\log(X_{n-1,n}) - \log(X_{n-k,n})]^r, \; r = 1, 2 \tag{15}$$

The estimators of c and $\hat{a}$ are then

$$\hat{c} = M_n^{(1)} + 1 - \frac{1}{2\{1 - [M_n^{(1)}]^2/[M_n^{(2)}]\}}; \; \hat{a} = uM_n^{(1)}/\rho_1 \tag{16, 17}$$

$$\rho_1 = 1, \; \hat{c} \ge 0; \; \rho_1 = 1(1 - \hat{c}), \; \hat{c} \le 0 \tag{18a,b}$$

The standard deviation of the asymptotically normal estimator of c is

$$\mathrm{s.d.}(\hat{c}) = [(1 + \hat{c}^2)/k]^{1/2}, \; \hat{c} \ge 0 \tag{19a}$$

$$\mathrm{s.d.}(\hat{c}) = \left\{[(1 - \hat{c})^2(1 - 2\hat{c})\left[4 - \frac{8(1 - 2\hat{c})}{(1 - 3\hat{c})} + \frac{(5 - 11\hat{c})(1 - 2\hat{c})}{(1 - 3\hat{c})(1 - 4\hat{c})}\right] \Big/ k\right\}^{1/2}, \; \hat{c} < 0 \tag{19b}$$

(de Haan 1994).

WIND-SPEED DATA

Uncorrelated samples obtained from largest daily data records: Sets of daily fastest mile wind speeds for winds blowing from any direction were obtained from the National Climatic Data Center, National Oceanic and Atmospheric Administration. In most samples a number of daily fastest miles were missing. The speeds on days with missing fastest mile data were estimated from speeds recorded on the respective days at 3-h intervals, using observations of the approximate relation between these speeds and daily fastest mile speeds. Wind speeds so estimated exceeded 15.6 m/s (35 mph) only at the following stations and dates: Boise, Idaho (April 26, 1987, 16.1 m/s); Portland, Oregon (November 14, 1981, 19.7 m/s), Salt Lake City, Utah (February 1, 1987, 16.1 m/s), and Toledo, Ohio (February 6, 1986, 18.8 m/s). Forty-four samples were used in the analyses. For 14 of these samples corrections based on the largest yearly records were made. Details on these corrections are given in (Simiu and Heckert 1995). The influence of these corrections on results of the analyses is discussed in the following section. The length of the records ranged from 15 to 26 years, the average length being about 18.5 years.

The anemometer elevations were changed during the period of record at the following stations: Duluth [16.2 m (53 ft) to October 15, 1975, 6.4 m (21 ft)] thereafter], Dayton [6.1 m (20 ft) to February 4, 1964, 7.7 m (22 ft) thereafter], Missoula [6.1 m (20 ft) to June 24, 1982, 9.8 m (32 ft) thereafter], Oklahoma City [16.8 m (55 ft) to October 21, 1965, 6.1 m (20 ft) thereafter], Portland [7.6 m (25 ft) to March 1, 1973, 6.1 m (20 ft) thereafter], San Diego [3.1 m (21 ft) to August 13, 1969, 6.1 m (20 ft) thereafter], Toledo [6.1 m (20 ft) to November 1, 1968, 10 m (33 ft) thereafter], and Winnemucca [10.4 m (34 ft) to April 22, 1966, 6.1 m (20 ft) thereafter]. For these stations the daily data were corrected to correspond to a common 10-m elevation using the logarithmic law for open terrain. For all other stations the anemometer elevations did not change during the period of record and (except for Denver, where the data were also corrected to correspond to a 10-m elevation) the original recorded data were used, that is, no elevation correction was effected.

From samples of largest daily wind speeds we obtained, as follows, samples that have a reduced mutual dependence among the data. Partition the sample of daily maximums into small periods of size equal to or larger than the duration of typical storms in days. [Based on statistical tests reported by Thom (1964), a reasonable choice of the length of the period is four to eight days.] Pick the largest value in each period. If the maximums of two adjacent periods are less than half a period apart, replace the smaller of the two maximums by the next smaller value in the respective period, which is at least half a period apart from the larger maximum. A data sample is thus obtained, in which adjacent data are one period apart on the average and never less than half a period apart. Following are the daily maximums at Boise, Idaho in the first six eight-day periods in 1965. The periods are separated by vertical bars. The data selected by the procedure just described are in bold type. In the sixth period we underlined the period maximum, 26, discarded and replaced it by the next largest value, 18, because of the proximity to the larger maximum, 31, of the adjacent period.

23, 32, **35**, 20, 26, 24, 24, 14|13, **16**, 5, 11, 5, 12, 12, 7|6, 6, 9, 9, 11, 12, 25, **26**|15, 12, 12, 7, 15, 12, **29**, 10|7, 10, 15, 20, 20, 17, 24, **31**|<u>26</u>, 9, 16, 14, **18**, 16, 14, 12|

In spite of our selection procedure, small correlations among data might subsist. Nevertheless, we refer to a sample obtained by the selection procedure just described as an uncorrelated data sample based on eight-day (four-day) intervals or, for short, an eight-day (four-day) interval sample. An assessment was made of differences between results of analyses based on four-day- and eight-day-interval samples at the same station. Since in all cases the differences were insignificant, we present in this report only results based on four-day-interval samples.

Fig. 1 contains typical histograms of the full samples of daily data and of the four-day-interval samples obtained from them for each of the 44 stations. Histograms for all 44 stations are included in Simiu and Heckert (1995). Owing to the small scale of the graphs, in some cases high wind data are not perceptible on the daily data histograms; however, they can be seen on the four-day-interval data histograms. A comparison between the histograms of the full daily data samples and the histograms of the four-day-interval samples shows that our selection procedure considerably reduces the number of lowest wind-speed data. The selection procedure also results in a shifting of the highest ordinate of the histogram toward higher wind speeds.

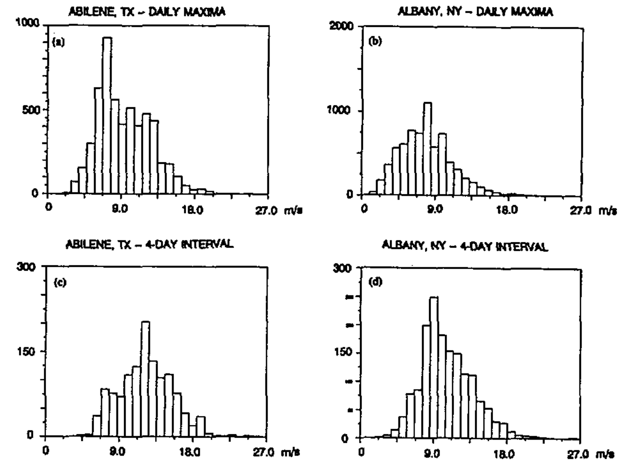

FIG. 1. Typical Histograms of: (a,b) Daily Largest Wind Speeds; (c,d) Four-Day-Interval Uncorrelated Wind Speeds

Largest yearly data samples: Also available were samples of largest yearly fastest miles for winds blowing from any direction, recorded over periods of 18 to 54 years at 115 U.S. stations not subjected to mature hurricane winds. The data in those samples were obtained and checked against original charts by M. J. Changery, Chief, Applied Climatology Branch, National Climatic Center, National Oceanic and Atmospheric Administration (personal communication December 20, 1988) and are an update of the information included in Simiu et al. (1979).

As noted earlier, all the data samples used in our analyses are available in an anonymous data file. Instructions for accessing the file and programs for generating uncorrelated data sets are given in Appendix I.

ANALYSES AND RESULTS

Analysis of uncorrelated data samples by the probability plot correlation coefficient method: Before applying the peaks over threshold approach, we estimated the best-fitting distributions for the four-day samples from among a set of seven distributions or families of distributions (normal, double exponential, lognormal, Gumbel, Fréchet, Weibull, and reverse Weibull). This analysis was viewed as a tentative step toward understanding the probabilistic structure of the populations from which the threshold exceedances were taken. The estimation of the best-fitting distribution was based on the probability plot correlation coefficient (PPCC) (Filliben 1975). As an example, Fig. 2 shows the PPCC plots for the Albany, New York four-day-interval samples. For this sample the mean and standard deviation were $E(X) = 10.5$ m/s (23.5 mph) and standard deviation (s.d.) $(X) = 3.14$ m/s (7.03 mph); for the full sample of daily data $E(X) = 7.82$ m/s (17.5 mph) and s.d. $(X) = 3.12$ m/s (6.98 mph).

The reverse Weibull distribution was found to best-fit the data in majority of the cases. Even in the cases where other distributions fitted the data better, the reverse Weibull was typically very close to being the best-fitting distribution, that is, its PPCC differed only in the fourth or even fifth significant figure from the PPCC of the best-fitting distribution. Therefore, we reanalyzed the eight-day-interval samples by assuming that the populations for all stations have reverse Weibull distributions with a single, site-independent value of the tail-length parameter, and site-dependent location and scale parameters. For each station we calculated the PPCCs by assuming that the shape parameter γ was 1, 2, 3, . . . , 50. For samples of data based on eight-day intervals the mean and median of the PPCCs, taken over all the stations, were largest for $\gamma = 11$ and $\gamma = 13$, respectively. If it were true that a reverse Weibull

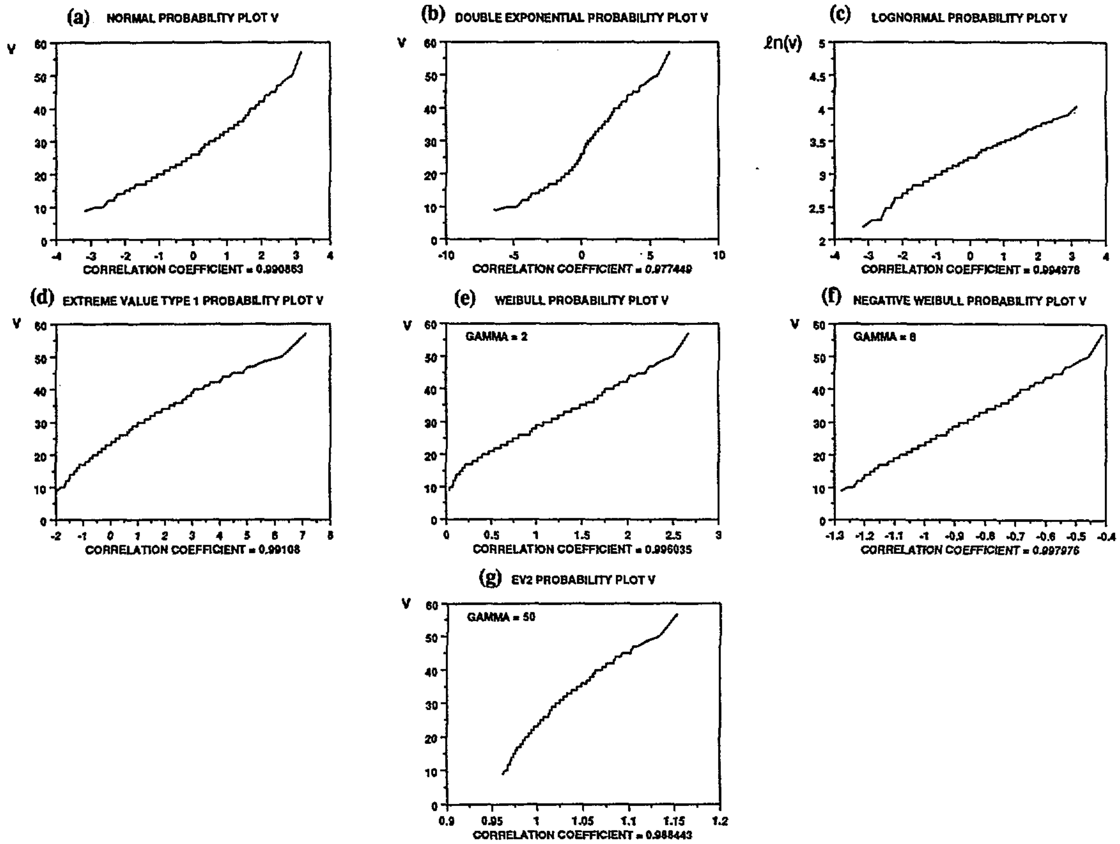

FIG. 2. Probability Plots for Nondimensionalized Speeds V (Four-Day-Interval Sample, Albany, New York)

distribution with a single tail-length parameter characterized the extreme winds at all sites, then our analyses would indicate that the value of that parameter is $\gamma \approx 12$.

The assumption that there exists a universal tail-length parameter for extreme wind distributions is implicit in current practice, except that it is applied to the Gumbel distribution (for which $\gamma = \infty$). To see whether that assumption is tenable if applied to the reverse Weibull distribution with $\gamma \approx 12$, 44 samples corresponding to 18-yr record lengths based on 8-day intervals were generated from reverse Weibull populations with (1) $\gamma = 8$, (2) $\gamma = 12$, and (3) $\gamma = 16$. The number of simulated samples for which the best-fitting reverse Weibull distribution had shape parameters with $\gamma \leq 12$, $13 \leq \gamma \leq 20$, and $\gamma \geq 21$ are shown in Table 1. Also shown in Table 1 are the numbers of observed samples based on 8-day intervals for which the analysis yielded $\gamma \leq 12$, $13 \leq \gamma \leq 20$, and $\gamma \geq 21$. The results of Table 1 would suggest that a reverse Weibull distribution with $\gamma \approx 12$ is an appropriate model for populations of extreme winds representing data based on 8-day intervals, except for the larger number of samples with $\gamma \geq 20$ among the observed samples than among the simulated samples. We interpret this larger number as reflecting the relatively frequent presence of outliers among the observed samples. This may suggest that, because wind-speed populations are mixed (in addition to extremes they include ordinary winds whose meteorological structure may differ from that of the extremes), a sample taken from such a population is not likely to be a sound basis for inferences on extremes. It is therefore desirable to "let the tails speak for themselves." The application of the GPD-based peaks over threshold approach is an attempt to do just this.

Estimation of tail-length parameter by peaks over threshold analyses of uncorrelated data samples: We applied the de Haan estimation method to the four-day-interval samples using, for each sample, a highest threshold such that the number of its

TABLE 1. Comparison of Results for Simulated and Observed Samples

Simulated samples (1)	$\gamma \leq 12$ (2)	$13 \leq \gamma \leq 20$ (3)	$\gamma \geq 21$ (4)
$\gamma = 8$	44	0	0
$\gamma = 12$	26	15	3
$\gamma = 16$	7	22	15
Observed samples[a]	25	10	9

[a]Stations for which $\gamma \geq 21$ were: Green Bay, Greensboro, Huron, Lansing, Louisville, Macon, Moline, Portland, or, San Diego. Those for which $13 \leq \gamma \leq 20$ were: Binghamton, Fort Smith, Fort Wayne, Grenville, Milwaukee, Minneapolis, Springfield, Topeka, Tucson, and Yuma.

exceedances be equal to, or larger than and as close as possible to, 16; any higher threshold was deemed to result in data sets too small to yield useful statistics. Denoting a sample's maximum threshold by u_{max}, the next higher thresholds we considered were $u_{max} - 1, u_{max} - 2, \ldots, u_{max} - 24$. Estimated values (point estimates) of c are shown for typical stations in the plots of Fig. 3. Also shown on the plots are 95% confidence bounds [i.e., lines corresponding to $\hat{c} \pm 2$ s.d.$(\hat{c})$]. On the horizontal coordinate axis of each plot we indicate the thresholds, in miles per hour, and the size of the data samples (i.e., the number of exceedances) for each threshold.

Fig. 4 is an example of a similar plot ($\hat{c}$ versus number of threshold exceedances) presented for a different type of extreme value problem by de Haan (1990). Like the plots in Fig. 3, this plot exhibits fairly strong fluctuations in the region of the highest thresholds where the sample size is relatively small. In the region of the smaller thresholds the 95% confidence bounds become narrower—a result of the increasing sample size—but a bias sets in, which is due to the inclusion in the data samples of data not properly belonging to the tails.

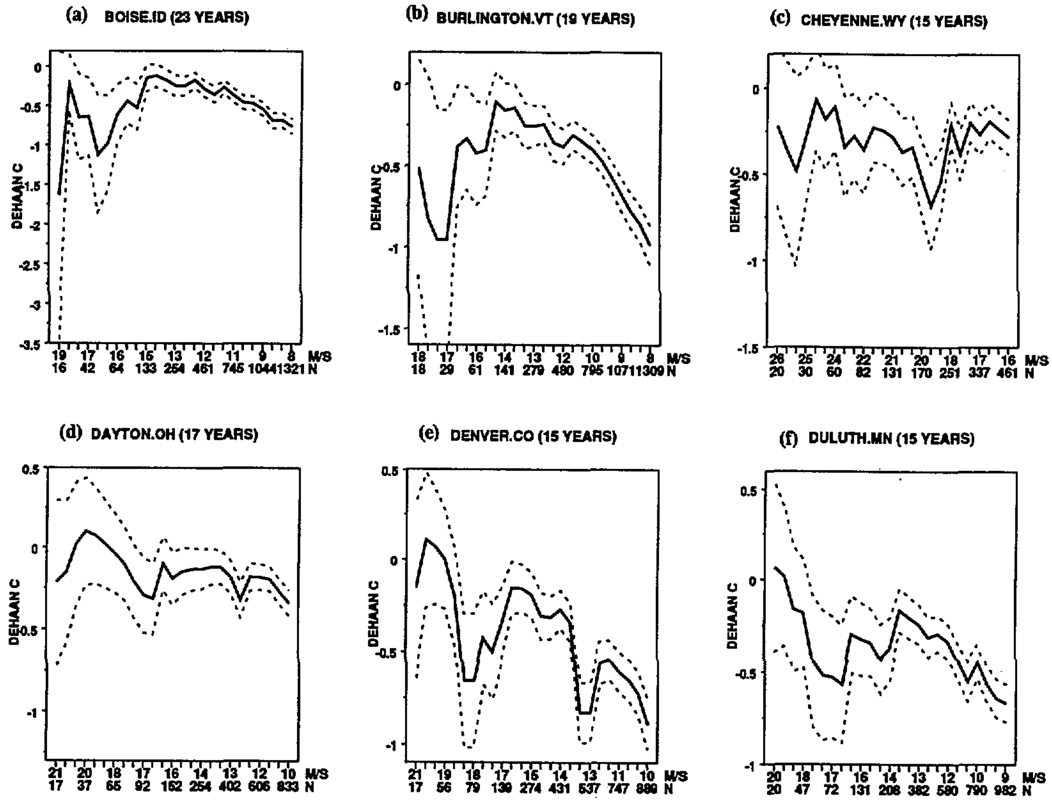

FIG. 3. Typical Plots of Estimate of Parameter *c* and 95% Confidence Bounds versus Threshold Speed (in m/s) and Number of Threshold Exceedances *N* (Based on Four-Day-Interval Samples)

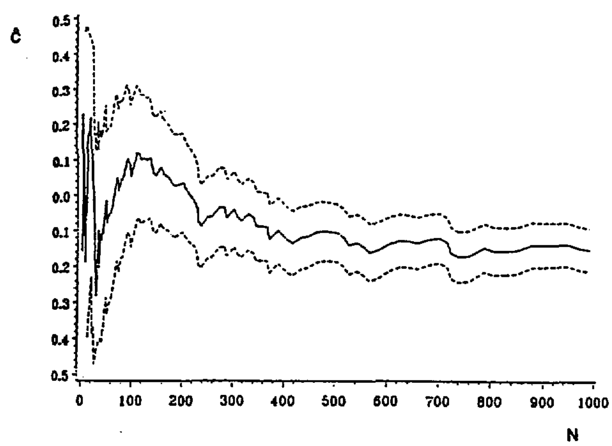

FIG. 4. Point Estimates of Tail-Length Parameter *c* and 95% Confidence Bounds versus Number of Exceedances *N* (de Haan 1990)

In de Haan's (1990) judgment: "It looks from the graph as if the value $c = 0$ is not a bad choice in this case."

We propose to apply this type of judgment to the plots in Fig. 3. For example, it would appear that, for Boise, $c < 0$, perhaps $c \approx -0.20$. The plots of Fig. 3 and similar plots in Simiu and Heckert (1995) indicate that $c < 0$ for most, though not all, stations. This is an interesting result in itself, insofar as it would indicate that in most cases extreme wind distribution tails are indeed finite.

Let us again assume for a moment that extreme wind speeds in regions not subjected to mature hurricanes are described by a reverse Weibull distribution, with site-dependent location and scale parameters and a site-independent tail-length parameter c. The weighted mean of c may be written as a function of threshold order q as (Gross et al. 1995)

$$\hat{c}_{wq} = \left\{ \sum_{i=1}^{44} \hat{c}_{iq}/s_{iq}^2 \right\} \Big/ \sum_{i=1}^{44} 1/s_{iq}^2 \tag{20}$$

where the index $q = 1, 2, \ldots, 25$ is the order of the highest, second highest, . . . , 25th highest threshold for the 44 samples being analyzed; and $\hat{c}_{iq}$, s_{iq} = estimated value of c and the estimated standard deviation of c for station i, based on the threshold of order q. (Recall that the threshold corresponding to $q = 1$ for each station was chosen so that at least 16 data points exceed that threshold.) The plot of c_{wq} is shown in Fig. 5 and, in our opinion, tends to confirm the view that, at most if not all stations, the estimated value of c is negative, perhaps $c \approx -0.2$ or $c \approx -0.25$. As suggested by Monte Carlo simulations (Gross et al. 1994), for sample sizes not exceeding about 10% of the total number of data, the bias in the estimation of c is about -0.05, that is, sufficiently small not to invalidate our judgment that (predominantly), $c < 0$.

Estimation of tail-length parameter by peaks over threshold analyses of largest yearly data samples: Fig. 6 includes point estimates of c for typical stations for which largest yearly speeds were available. Also shown on the plots are 95% con-

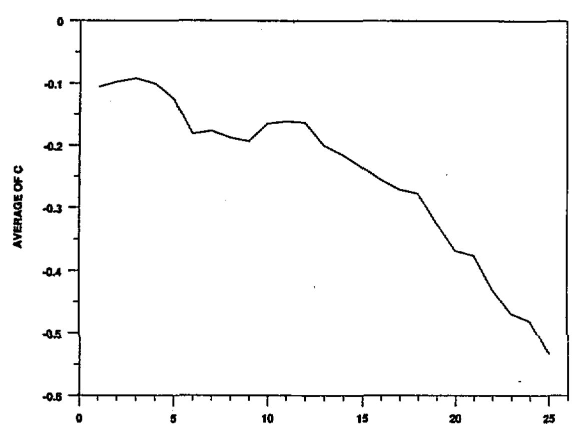

FIG. 5. Mean of Estimates of Tail-Length Parameter *c* Weighted over 44 Four-Day-Interval Samples versus Order of Threshold

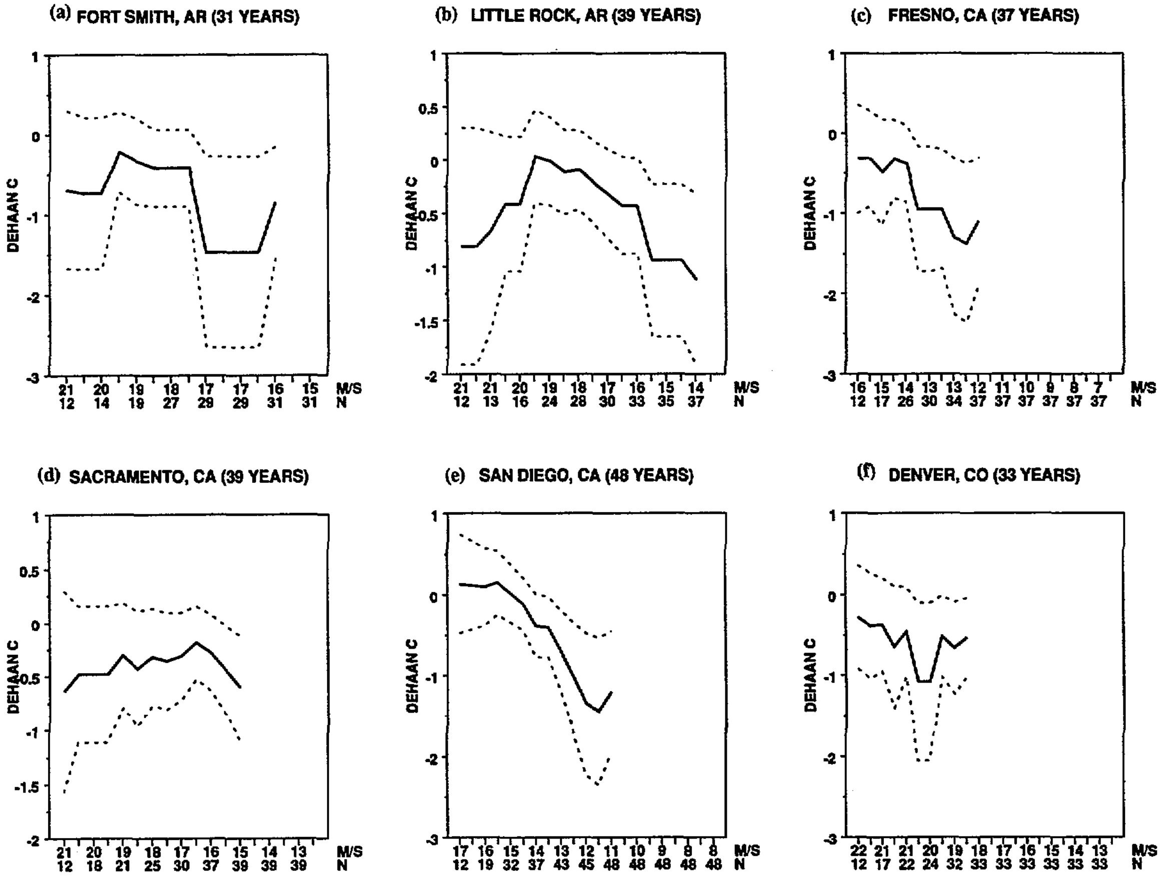

FIG. 6. Typical Plots of Estimates of Parameter *c* and 95% Confidence Bounds versus Threshold and Number of Threshold Exceedances *N* (Based on Samples of Largest Yearly Speeds)

fidence bounds [i.e., lines corresponding to $\hat{c} \pm 2$ s.d.$(\hat{c})$]. The estimates are plotted against the threshold speed and the number of exceedances of the threshold, as in Fig. 2. For these plots the larger samples are not likely to be affected by bias, since the lowest wind speed in those samples is itself a largest yearly wind, and hence it will be within or close to the distribution tail. Though the plots are not always easy to interpret, in our opinion they confirm the view that at most stations c is negative. Fig. 7, which shows the weighted average of the estimated tail-length parameter for the 115 data samples [(20)], lends further credence to this view.

Estimation of wind speeds with specified mean recurrence intervals by peaks over threshold analysis of correlated data samples: Fig. 8 contains plots of point estimates of the extreme winds with mean recurrence intervals of 100, 1,000, and 100,000 years. The estimates were based on four-day-interval samples at each of the 44 stations. They are plotted against the threshold speed and the number of exceedances of the threshold, as in Fig. 2.

We reproduce in Fig. 9 an example of a plot where the quantile fluctuates strongly as a function of threshold (de Haan 1990). De Haan comments: "If one would be forced to give a point estimate a value of 510 cm . . . would not be unreasonable." The comment is indicative of the spirit in which results based on the peaks over threshold method must be interpreted in cases where fairly large fluctuations are present, as is the case for Fig. 9 and many of the plots in Fig. 8. We do not attempt in this report to estimate extreme wind speeds for various mean recurrence intervals. Rather, having found that the tail-length parameter c of the GPD is negative for majority of the stations, we assess in the following section the potential implications of this finding for the estimation of load factors.

Influence of data errors on analysis results: Simiu and Heckert (1995) list errors in the recorded daily data discovered at 14 stations, and the respective corrected values based on an examination of original traces at those stations. Plots of estimated c values were found to be generally weakly influenced by such errors. However, speeds with various mean recurrence intervals were in some cases affected fairly significantly by the errors in data. The plot of the weighted mean over all 44 stations, computed from results obtained by using the uncorrected data, was indistinguishable from Fig. 5.

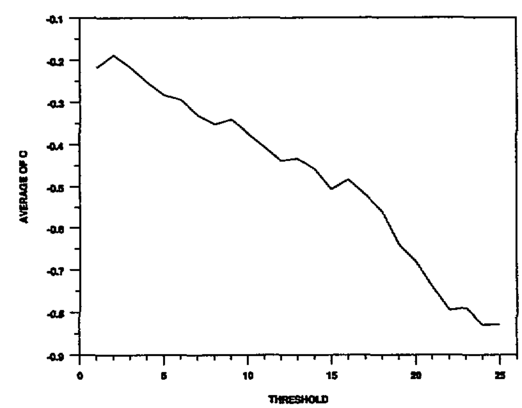

FIG. 7. Mean of Estimates of Tail-Length Parameter *c* Weighted over 115 Largest Yearly Data Samples versus Order of Threshold

LOAD FACTORS FOR WIND-SENSITIVE STRUCTURES

Extreme wind loads used in design include nominal basic design wind loads (e.g., the 50-yr wind load) and nominal ultimate wind loads. A basic design wind load is an extreme

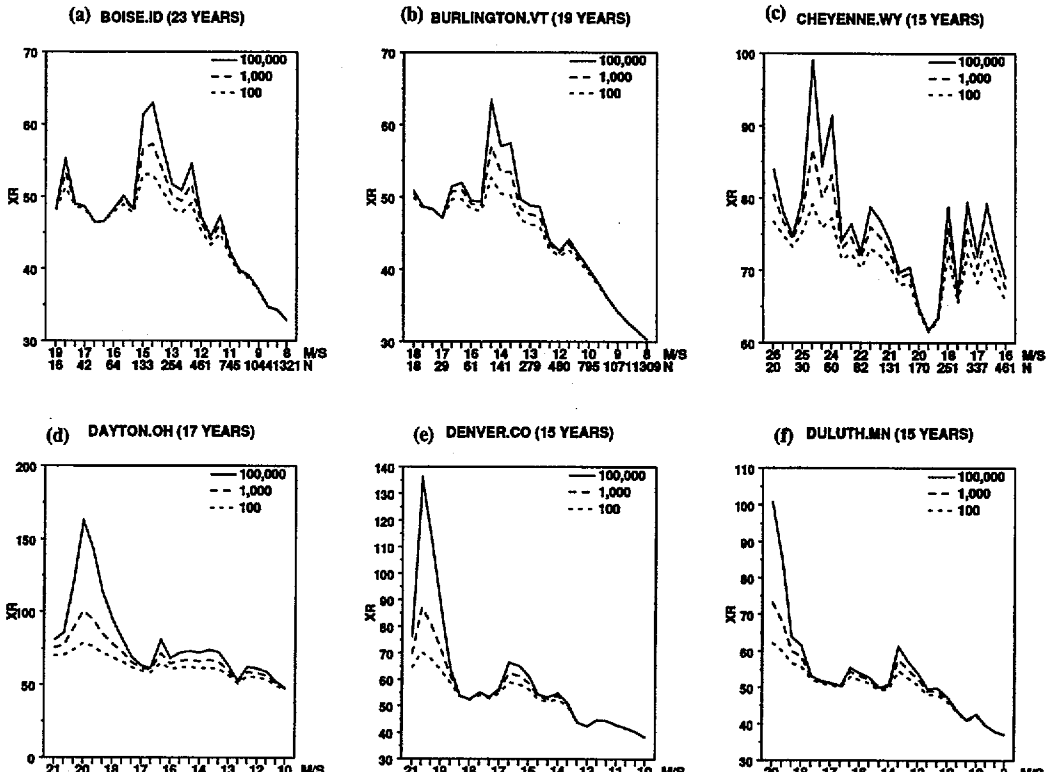

FIG. 8. Typical Plots of Point Estimates for 100-yr, 1,000-yr, and 100,000-yr Speeds versus Threshold and Number of Threshold Exceedances *N* (Based on Four-Day-Interval Samples)

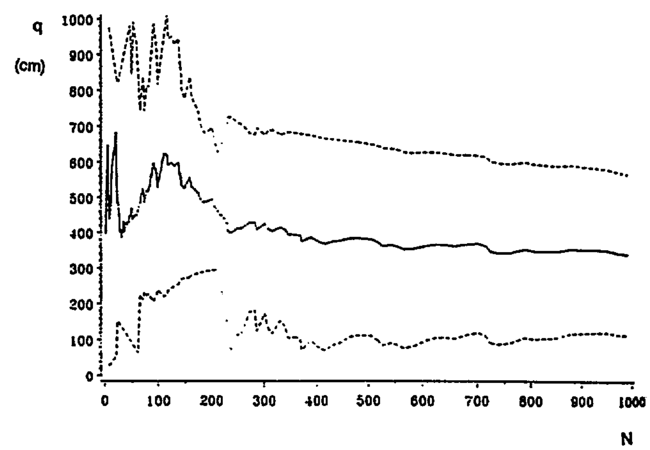

FIG. 9. Point Estimates and 95% Confidence Bounds for Quantile *q* (in cm) versus Number of Exceedances *N* (de Haan 1990)

load with the specified probability of being exceeded during a basic time interval. In the United States that interval is usually 50 years. A basic design load with a 50-yr mean recurrence interval has a probability of almost two-third of being exceeded during a 50-yr period.

A structure or element thereof is expected to withstand loads substantially in excess of a 50- or 100-yr wind load without loss of integrity. The wind load beyond which loss of integrity can be expected is referred to as the nominal ultimate wind load. The nominal ultimate strength provided for by the designer is based on a nominal ultimate wind load equal to the basic design wind load times a wind load factor. This statement is valid for the simple case where wind is the dominant load. It needs to be modified if load combinations are considered, but for clarity we refer here only to this case.

The load factor should be selected so that the probability of occurrence of the nominal ultimate load is acceptably small. This probabilistic concept is important from an economic or insurance point of view. To the extent that evacuation or similar measures cannot be counted on to prevent loss of life, it is also important from a safety point of view.

A probabilistic approach has proven helpful in a number of cases, particularly for relative assessments of alternative design provisions, for example, for mobile homes. However, in most cases the difficulties of obtaining wind load factors by probabilistic methods have proven to be substantial if not prohibitive. For this reason code writers have largely relied on wind load factors implicit in traditional codes and standards. For example, the ASCE *Standard A7-93* (1993) specifies a wind load factor of 1.3. In a very large number of applications the wind load is proportional to the square of the wind speed, so that a basic design wind speed and a nominal ultimate wind speed may be defined, which are proportional to the square root of the basic design wind load and the square root of the nominal ultimate wind load, respectively. For example, for Lander, Wyoming, the *ASCE Standard A7-93* (1993) specifies a basic design 50-yr design speed of 35.8 m/s (80 mph) (fastest mile at 10-m elevation). The corresponding nominal ultimate wind speed would then be $1.3^{1/2}35.8 = 40.8$ m/s (91.2) mph.

Reliance on traditional code values is part of the process sometimes referred to as "calibration against existing practice." Traditional codes were generally adequate for many types of structures, but questions remain on whether safety margins implicit in those codes may be applied to modern structures, which can differ substantially from their predecessors in their materials and design/construction techniques. For this reason an assessment of wind load factors used in codes and standards would be desirable. For example, one would wish to answer the question: what is the approximate mean recurrence interval of the nominal ultimate wind speed?

The answer to this question depends strongly on the probability distribution assumed to best-fit the extreme wind speeds. For example, a PPCC analysis of largest yearly fastest-mile speeds recorded at Denver between 1951 and 1977, based on the assumption that the best-fitting distribution is Gumbel, yielded a 27.9 m/s (62.3 mph) estimate of the 50-yr wind speed at 10 m above ground. The corresponding nominal ultimate wind speed would be $1.3^{1/2}27.9 = 31.8$ m/s (71.0 mph), to which there would correspond, under the Gumbel assumption, a mean recurrence interval of about 500 years (Simiu et al. 1979). If taken at face value this would be an alarmingly short recurrence interval, since it would entail an unacceptably large probability of exceedance of the nominal ultimate wind load during the life of the structure.

However, the 500-yr mean recurrence interval is based on the Gumbel model. Since our results support the assumption that (predominantly) the appropriate model is a reverse Weibull distribution, rather than a Gumbel distribution, we wish to answer the question: what is the mean recurrence interval corresponding to $1.3^{1/2}$ times the wind speed with a 50-yr mean recurrence interval? For Denver, if one had to estimate the tail-length parameter $\hat{c}$ from the plots of Figs. 3 and 6, and the 50-yr speed from the plot of Fig. 8, one might choose $\hat{c} = -0.2$ (a conservative choice: according to the plots $\hat{c}$ is likely to be somewhat lower, that is, the distribution tail is likely to be somewhat shorter than that corresponding to $c = -0.2$), and $x_{50} = 26.8$ m/s (60 mph). For a threshold of 16.5 m/s (37 mph)—a value that is roughly consistent with these choices (see Denver plot, Fig. 3)—we have $\lambda = 139/15 = 9.27$/yr. Assuming $x_{50} = 26.8$ m/s (60 mph), it would follow from (13) and (14) that $\hat{a} = 2.5$ m/s (5.64 mph). The estimated maximum possible wind speed corresponding to the parameters $\hat{c} = -0.2$ and $\hat{a} = 2.5$ m/s (5.64 mph) is obtained by letting $R \to \infty$ in (13), (14). Its value is $x_{max} = u - \hat{a}/\hat{c} = 29.1$ m/s (65.2 mph). The estimated mean recurrence interval of the nominal ultimate wind speed $1.3^{1/2}x_{50} = 1.3^{1/2} \times 26.8 = 30.6$ m/s (68.4 mph) is therefore infinity (i.e., such a wind speed is estimated to never occur).

This estimate is of course subject to sampling errors: the actual maximum possible wind speed may be higher than 29.1 m/s (65.2 mph), and the mean recurrence interval of the 30.6 m/s (68.4 mph) speed may in fact be finite, though likely much longer than 500 years. In spite of the uncertainties inherent in our estimates, our result suggests that a load factor of 1.3—specified in ASCE standard on the basis of practical experience—is in fact reasonably adequate from a probabilistic point of view. This is contrary to what would be concluded if the analysis were based on the assumption that the Gumbel distribution holds.

Since the probabilistic model of extreme wind speeds suggested by our results tends to be consistent with experience of long standing incorporated in standard provisions for wind loads, its prudent adoption should help remove doubts still persisting among some practitioners on the usefulness and adequacy of probabilistic approaches to the development of credible wind loading criteria. However, for this to be the case, the reliability framework used for such development should make proper allowance for the uncertainties inherent in estimates of the extreme wind-speed distribution parameters.

CONCLUSIONS

We presented estimates of the tail-length parameters of extreme wind distributions for nontornadic winds blowing from any direction in regions unaffected by mature hurricanes. In our opinion, these estimates support the view that the reverse Weibull distribution is an appropriate probabilistic model in most if not all cases. They also suggest that load factors for wind sensitive structures specified by current standards provide for reasonable safety margins against wind loads, and that the adoption of the Gumbel model likely results in an unrealistic assessment of structural reliability under wind loads.

However, owing to fluctuations of our estimates with the threshold value, it is difficult to provide reliable quantitative estimates of the tail-length parameters. This difficulty is even more pronounced for quantile estimates. We tentatively ascribe these difficulties to the relatively small size of our samples (15–26 years). It would therefore be desirable to assemble data for longer records than those used in this paper. In addition, more efficient estimation methods should be developed, if possible.

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APPENDIX I.

Instructions for Accessing Data Sets and Attendant Programs

Note: Only corrected data are included in the data files.

- ftp enh.nist.gov (or: ftp 129.6.16.1)
- >user anonymous
- enter password >guest
- >cd emil/datasets (to access data)
- >cd emil/programs (to access programs)
- >prompt off
- >mget * (this copies all the data files)
- >dir (this lists the available files)
- >get <enh name> <local name> (this copies a specific file; example: get boise.id boise.id)
- >quit

APPENDIX II. REFERENCES

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