Taming chaotic dynamics with weak periodic perturbations: an elucidation and critique

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Taming chaotic dynamics with weak periodic perturbations: an elucidation and critique

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Abstract
It has been pointed out that a chaos control technique wherein a *weak* excitation is added to the system could be useful for the control of friction at the nanoscale. We examine the claim that this technique can achieve chaos control even though the added excitation is weak. We show that this claim is only valid for the particular case of systems with very shallow potential wells. However, the applicability of the technique is more general provided that the added periodic force is not restricted to being weak.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Efforts to create nano-devices and new ultra-small technology capable of improving the performance of robots, computers, communications and other electro/optical/mechanical devices must account for the role of friction. For this reason control of friction, and in particular control of chaos associated with its presence, is considered to be an important research area within the broad framework of manipulation of properties at nanoscale [1]. In this context, for the control procedure to be useful, it is important that the external perturbations be weak. According to [1, p 17], one of the techniques applicable for chaos control is the addition to the system of 'tiny external perturbations', as proposed in [2].

The technique proposed in [2] and results obtained therein are also mentioned in [3, p 605] and [4]. The latter notes that 'there is not sufficient rigorous theory to support this approach'. In this paper we show that it is nevertheless possible to assess the applicability, and point out a significant limitation of this technique.

2. Discussion

The system used in [2] to demonstrate the proposed control technique is a Josephson junction and therefore belongs to the class of planar multistable systems that possess a Melnikov function. The necessary condition for the occurrence of chaos in a dissipative planar multistable system with forcing $A\cos(\omega t) + a\cos(\beta\omega t + \phi)$ is that its Melnikov function $M(t)$ have simple zeros [5, 6] and [7, (section 2.5.3)], where, for each potential well,

$$M(t) = -k + A\alpha(\omega)\cos[\omega t + \vartheta(\omega)] + a\alpha(\beta\omega) \times \cos[\omega t + \vartheta(\beta\omega) + \varphi], \quad (1)$$

the constant k depends on the dissipation term in the system's equation of motion and (via the associated homoclinic or heteroclinic orbit) on the shape of the well, and $\alpha(\omega)$, $\vartheta(\omega)$, termed the Melnikov scale factor and the Melnikov phase angle, respectively, depend on the shape of the well. Strictly speaking, the Melnikov necessary condition for chaos is valid if the dissipation and excitation terms in the equation of motion of the system are asymptotically small, but it has been amply demonstrated in the literature that it also holds for sufficiently small perturbations of interest in practical applications. Associated with the Melnikov function is the functional known as the phase space flux factor Φ [6, 7], a measure of the degree to which the system is chaotic. If the Melnikov function has no simple zeros the factor Φ is zero. If the Melnikov function has simple zeros, Φ increases as a increases; for fixed a, Φ increases as the frequency excitation $\beta\omega$ is closer to the frequency for which the Melnikov scale factor is largest [7–10]. As Φ increases the system becomes *more* chaotic. The effect of the added excitation is therefore to intensify, rather than tame, the chaos. This is the case for

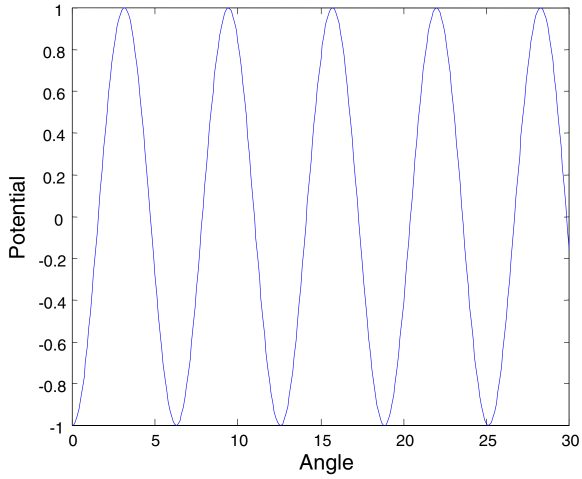

Figure 1. System potential.

$\Phi > 0$ and $a < a_1(A, \omega, \beta)$, where $a_1(A, \omega, \beta)$ is a threshold above which Melnikov theory breaks down. (For details on dependence on A, ω and β, see, e.g., [8].)

For a larger than a threshold $a_2(A, \omega, \beta)$ that can be determined experimentally or numerically, a transition occurs from chaotic motion to periodic (or, if ω and $\beta\omega$ are incommensurate, quasiperiodic) rotational motion. For the interval $a_1(A, \omega, \beta) < a < a_2(A, \omega, \beta)$, depending upon the structure of the system's bifurcations, it may be expected that as a increases, the motion becomes less chaotic; if windows of periodicity exist, the motion may become periodic or quasiperiodic. However, the absence of a theory, noted in [3], means that no dependable guidance is available for practical applications such as, for example, the control of chaos associated with friction at the nanoscale. However, transition from chaos to rotation definitely occurs for $a > a_2(A, \omega, \beta)$, and this fact can be used in practice for the purpose of chaos suppression.

The objective of this paper is to show that, in general, the value of a required to achieve chaos suppression is not small in relation to A, as is stated in [1] and [2]. The role of the added excitation is to supply the complementary energy

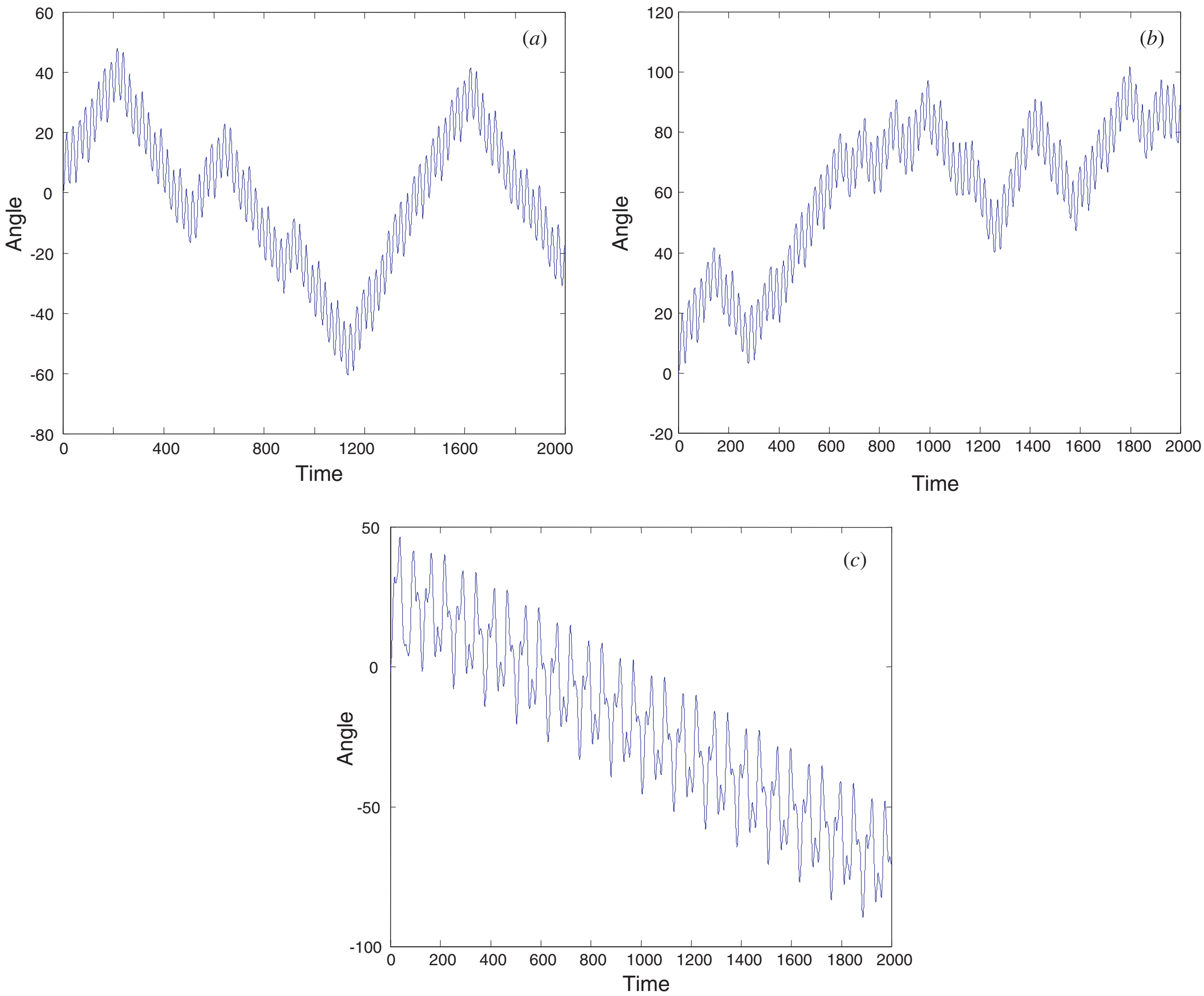

Figure 2. (*a*) Chaotic motion ($a = 0$, $A = 1.75$, $G = 0.7$, $I = 0$). (*b*) Chaotic motion ($a = 0.0125$, $\beta = 0.4$, $A = 1.75$, $G = 0.7$, $I = 0$). (*c*) Periodic rotational motion ($a = 1.0$, $\beta = 0.4$, $A = 1.75$, $G = 0.7$, $I = 0$).

needed to bring about the transition from chaos to rotation. For the case of multistable systems—the case discussed in [2]—the added forcing needed to cause the transition is weak only if the system's potential wells are shallow. If this is not the case, the added periodic perturbation cannot be weak, as would be required, according to [1], for the control of chaos associated with friction at the nanoscale.

To explain the statements of the preceding paragraph we consider the system known as the Josephson junction used in [2] to demonstrate the technique proposed therein:

$$\ddot{\theta} + G\dot{\theta} + \sin\theta = I + A\sin(\omega t) + a\sin(\beta\omega t). \quad (2)$$

We assume, as in [2], $G = 0.7$, $\omega = 2\pi/25.12$. Let us first consider the case $I = 0$. The system potential for this case is shown in figure 1.

For $a = 0$ and $A = 1.75$, it can be verified that the behaviour of the system is chaotic (figure 2(a)). We now consider, as in [2], the case $a = 0.0125 \ll A$, $\beta = 0.4$. The system can again be verified to be chaotic (figure 2(b)). This is so because the potential energy associated with the height of the potential barrier is relatively large in relation to the energy associated with the total forcing, that is, that energy is not

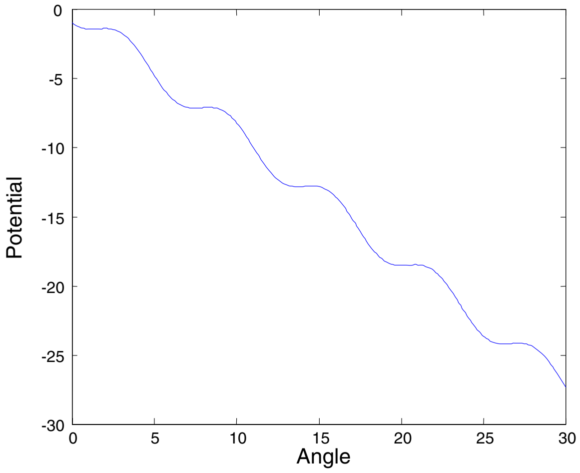

Figure 3. System potential, $I = 0.905$.

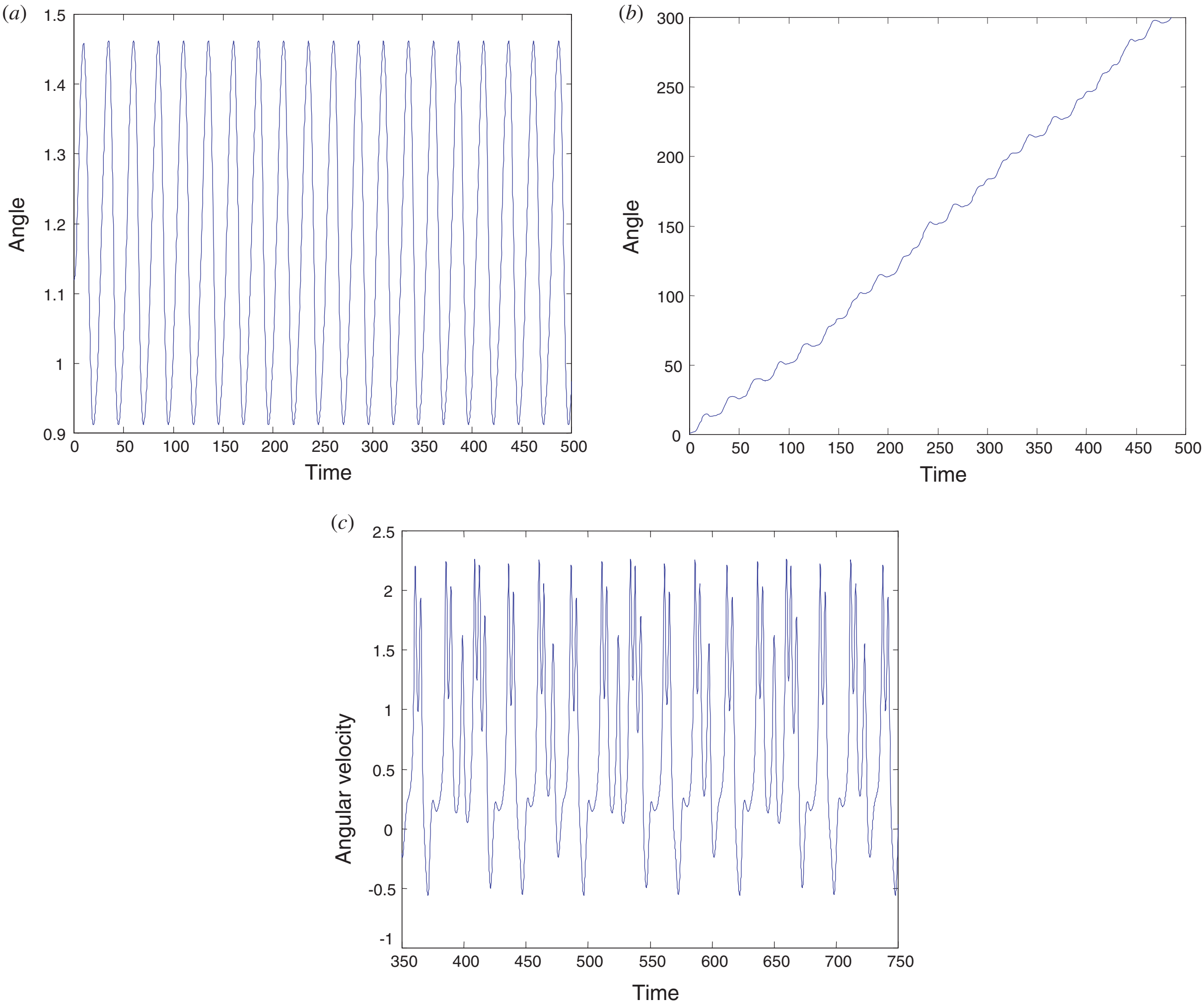

Figure 4. (a) Periodic librational motion ($a = 0$, $\beta = 0.4$, $A = 0.1$, $G = 0.7$, $I = 0.905$). (b) Chaotic motion ($a = 0$, $\beta = 0.4$, $A = 0.4$, $G = 0.7$, $I = 0.905$). (c) Periodic rotational motion ($a = 0.0125$, $\beta = 0.4$, $A = 0.4$, $G = 0.7$, $I = 0.905$).

sufficient to cause a transition from chaotic motion to rotational motion. However, for the larger added forcing $a = 1$, the system is no longer chaotic. Rather, its motions consist of periodic rotations—see figure 2(c).

We now consider, as in [2], the case $I = 0.905$. The potential, rather than having the expression $U(\theta) = -\cos(\theta)$, as was the case for $I = 0$, has the expression $U(\theta) = -\cos(\theta) - I\theta$ (figure 3). Figure 3 shows that the height of the potential barrier with respect to the bottom of the well located at its left is considerably less than was the case for $I = 0$. For $A = 0.1$ and $a = 0$, the steady state motion starting with zero initial velocity at the bottom of a well (i.e. at the point where the tangent to the potential curve vanishes) is periodic and librational (figure 4(a)). For $A = 0.4$, $a = 0$ the motion is chaotic (figure 4(b)). If we add harmonic forcing with $a = 0.0125$, the motion becomes periodic and rotational, that is, a transition from chaos to rotational motion will have occurred (figure 4(c)). That the amplitude a needed for the transition to occur is small is due to the fact that the potential barrier to be overcome is low; however, the mechanism is exactly the same in this case as in the case of figure 2(c).

3. Conclusion

We conclude that, for multistable planar systems, the statement in [1] and [2] that weak added periodic excitation may be used to tame chaotic behaviour is appropriate only for the particular case in which the height of the system's potential wells is low. Otherwise, the method proposed in [2] remains applicable provided, however, that the requisite added excitation, rather than being 'tiny', is relatively strong.

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