Smoke Movement in Corridors - Adding the Horizontal Momentum Equation to a Zone Model

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Introduction

The most common type of models utilized to study building fires are referred to as zone models. The motivation for using such models in preference to a complete implementation of the Navier Stokes equation is the great difficulty in obtaining solutions of the latter in realistic fire scenarios. One uses only a few elements, or zones, per compartment, and thus can apply the technique to many compartments. A more complete description of zone models is given elsewhere[1,2,3]. This type of model works well in many cases. However, for long rooms or tall shafts, the basic tenet of the finite element concept as applied to fires is violated. The idea is that within an element, or zone, the gases are uniformly mixed, and there is no bulk velocity of the gas. To simulate smoke movement in large buildings, it is important to predict movement of the smoke front(nose) in the corridor or shaft. Although the ideas which we will discuss are also applicable to tall shafts, there will be differences in both the equations, and their implementation. This paper concentrates on the flow along a horizontal corridor.

Benjamin[4] developed the treatment of steady gravity fronts for two fluids at constant density. Zukoski et. al.[5,6] have studied smoke spread by using salt water analogue experiments. These are also treatments of the flow of gas at a constant density. The latter also performed experiments to consider the effect of heat loss on the velocity of the gas. Heskestad[7] studied full scale experiment in a corridor off of a burn room, but the boundary condition of the inflow into the corridor is uncertain. Jones and Quintiere[8] developed an analysis to treat the smoke filling in a corridor by two-zone model. But their approach does not incorporate the transient problem of the movement of the smoke front(nose), and cannot predict the arrival of the front and the far end of a corridor.

The hybrid model is a new approach to considering a transient smoke flow in a corridor, and is intermediate between a zone model and field model. Since the intent of this hybrid model is to augment the classical zone model formulation, we derive the necessary equations in that same context. First we derive the equation necessary to model the finite velocity of the smoke layer. We discuss the various approximations which can be made and how they effect the agreement between experiment and theory. In particular, we discuss the effect of the pressure term by comparing the predictions when the pressure gradient is ignored to that when it is included. At present we do not consider the effect of heat transfer from the gas layer to the boundaries.

Hybrid Model

The hybrid model considers two zones, the upper layer (called a 'smoke layer') and a lower, cooler, layer (called 'air layer'). However, the important difference from the classical zone model is that all quantities in each layer depend upon the horizontal coordinate in the corridor direction.

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Reprinted from: Proceedings of 12th Joint Panel Meeting of the UJNR Panel on Fire Research and Safety, Oct. 27 - Nov. 2, 1992, Produced by: Building Research Institute, Tsukuba, Ibaraki and Fire Research Institute, Mitaka, Tokyo, 1994.

This requires consideration of the horizontal momentum balance. In effect, we have subdivided each zone in a manner similar to a field model, but only in one direction.

If we assume that there is no mass transfer vertically across zones, and that the gas layers are uniform across the width of the corridor, then the mass, momentum and energy equations can be integrated over the horizontal (y-direction) and vertical (z-direction) of each zone at any along the corridor. Each conservation equation (for mass, momentum and energy) can be written in a form similar to that of mass, which is

$$\int_0^B\int_{h_1}^{h_2}\frac{\partial\rho}{\partial t}dA+\int_0^B\int_{h_1}^{h_2}\frac{\partial\rho u}{\partial x}da+\int_0^B\int_{h_1}^{h_2}\frac{\partial\rho v}{\partial y}dA+\int_0^B\int_{h_1}^{h_2}\frac{\partial\rho w}{\partial z}dA=0 \tag{1}$$

The set is closed with the equation of state

$$P = \rho RT \tag{2}$$

The limits on the integral over the width (y) are from 0 to B since we have assumed uniformity over the width and the horizontal velocity in this direction is assumed to vanish. Eventually this restriction must be lifted, as when we wish to consider openings in the middle of the corridor. However, for the present, this approximation will suffice. The integral over height is from h_1 to h_2 since we need one of these for each layer, in this case two. For the upper layer, $h_1=0$, $h_2=h$, $\rho=\rho_a$, $u=u_a$, $v=0$, and $w=w_a$. For the lower layer, $h_1=h$, $h_2=H$, $\rho=\rho_s$, $u=u_s$, $v=0$, and $w=w_s$. h is the height of interface from floor and H the height of ceiling. This area of integration is $dA=dxdy$.

Simplifying Assumptions

We now consider this as a two dimensional problem, in the height and length only. We can simplify the problem somewhat based on the use of these equations which will be in the context of a zone model. They are as follows:

a) Uniform velocity distribution in both the height and width in each zone,
b) Hydraulic Pressure distribution in the vertical,
c) No entrainment along the boundary between the two layers,
d) Lower Layer temperature is the same as reference,
e) Ignore the effects of viscosity,
f) Consider the cases of the floor (reference) pressure to be either constant, or included by substituting the pressure gradient term into the momentum equation.

Basic Relations

The vertical velocity of the moving interface is

$$w_s=\frac{\partial h}{\partial t}+u_s\frac{\partial h}{\partial x},\ and\ w_a=\frac{\partial h}{\partial t}+u_a\frac{\partial h}{\partial x} \tag{3}$$

for the upper and lower elements, respectively. The chain rule for derivatives of integrals of the variables is

By expanding the integral of the derivative of the pressure term using the chain rule, and

$$\frac{\partial}{\partial t}\int_{a(x,t)}^{b(x,t)} f(x,t)dz = f(b)\frac{\partial b}{\partial t} - f(a)\frac{\partial a}{\partial t} + \int_{a(x,t)}^{b(x,t)} \frac{\partial f}{\partial t}dz \tag{4}$$

evaluating it assuming hydrostatic equilibrium in the vertical, we have the following exact solution for the pressure gradient from the original momentum equation:

$$\int_{h_1}^{h_2} \frac{\partial P}{\partial x}dz = \begin{cases} (H-h)\dfrac{\partial P_f}{\partial x} - (\rho_a-\rho_s)g(H-h)\dfrac{\partial h}{\partial x} & \textit{for the smoke layer} \\ h\dfrac{\partial P_f}{\partial x} & \textit{for the air layer} \end{cases} \tag{5}$$

The relation between the total pressure, P_f, and the relative pressure from a reference point, P_0, is $p_f=P_f-P_0$. Then we have the relation of a derivative for pressure as follows:

$$\frac{\partial P_f}{\partial x} = \frac{\partial p_f}{\partial x}. \tag{6}$$

Hybrid Model with no heat transfer

Using the assumptions and basic relations, we can obtain predictive equations for the horizontal momentum and vertical interface height for incompressible flow and no heat transfer. From mass conservation in the upper layer, the equation for the change of interface height, h, can be written as

$$\frac{\partial h}{\partial t} + u_s\frac{\partial h}{\partial x} - (H-h)\frac{\partial u_s}{\partial x} = 0. \tag{7}$$

Given conservation of total mass in the two layers, we have

$$\frac{\partial (H-h)u_s}{\partial x} + \frac{\partial h u_a}{\partial x} = 0. \tag{8}$$

Therefore the relation of mass conservation on arbitrary vertical plane is

$$(H-h)u_s + hu_a = V_0(t). \tag{9}$$

From momentum and mass conservation of smoke, the equation of change of velocity in the upper layer is

$$\frac{\partial u_s}{\partial t} - \frac{\rho_a-\rho_s}{\rho_s}g\frac{\partial h}{\partial x} + u_s\frac{\partial u_s}{\partial x} + \frac{1}{\rho_s}\frac{\partial P_f}{\partial x} = 0 \tag{10}$$

We have a similarly equation for the lower layer.

There are two possible approximations which can be made in order to solve eqs (10). The first is to ignore fluctuations in the floor or reference pressure, P_f because the effect of the change of pressure is smaller in comparing another term in building fires. When we ignore the effect of pressure in eqs (10) and ?, only eq (10) is used and eq ? is unnecessary. The equation for the velocity becomes

$$\frac{\partial u_s}{\partial t} - \frac{\rho_a - \rho_s}{\rho_s} g \frac{\partial h}{\partial x} + u_s \frac{\partial u_s}{\partial x} = 0. \tag{11}$$

We can describe u_a explicitly as a function of u_s from eq (9),

$$u_a = \frac{u_{s0}(H-h_0) - (H-h)u_s}{h}, \tag{12}$$

where u_{s0} is the inflow velocity and h_0 the interface height from floor at boundary x=0. We have two unknown variables, h and u_s. Therefore we can solve this system by using eq. (7) and (11), and calculate u_a explicitly by eq (12) under the boundary condition, $h=h_0$ and $u_s=u_{s0}$ at x=0.

In the second case we maintain the fluctuations in the reference pressure. For the hybrid model with no heat transfer, the unknown variables are h, u_s, u_a and P_f and the known quantities are ρ_s, ρ_a. We can simplify the system by eliminating the gradient P_f. When we eliminate the term $\partial P_f/\partial x$ by substituting eq ? into eq (10), the equation for the velocity of the upper layer becomes

$$A\frac{\partial u_s}{\partial t} + \left[-\frac{\rho_a - \rho_s}{\rho_s} g - (u_a - u_s)BC\right]\frac{\partial h}{\partial x} + \left[u_s + Cu_a\frac{H-h}{h} - BC(H-h)\right]\frac{\partial u_s}{\partial x} = 0$$
$$\text{where} \quad A = 1 + \frac{\rho_a(H-h)}{\rho_s h}, \quad B = -\frac{u_{s0}(H-h)}{h^2} + \frac{H}{h^2}u_s, \quad C = \frac{\rho_a}{\rho_s}. \tag{13}$$

u_a is the same as before, so we use eq (12) again. In this case, eq (7) and (13) are solved with the same boundary conditions as when the pressure term is ignored.

A Simple Model with Heat Transfer

The effect of heat transfer is very important for smoke movement in a corridor. At present we are not able to include the heat loss mechanism in the hybrid model. In order to estimate its effect, we have developed a simple model. Usually a zone model assumes the following relation between the velocity and the pressure difference across an opening,

$$\rho_s u_f^2/2 = \Delta P \tag{14}$$

and a uniform density in a zone. When we consider the hydrostatic pressure distribution in the vertical direction at the front of the corridor plume to be the same as at the opening, we have a following relation at the nose of the smoke flow:

$$\Delta P = \Delta\rho g z \tag{15}$$

where $\Delta\rho=\rho_a-\rho_s$.

From eq (14) and (15) we can obtain an equation for the velocity of the smoke at any height above the interface. It is

$$u_f = (2\Delta\rho g z/\rho_s)^{1/2}, \tag{16}$$

and the volumetric flow rate at the nose of the jet, Q_f, is

$$\begin{aligned} Q_f &= B\int_0^{h_f} u_f dh \\ &= \frac{2}{3}B\left(\frac{2\Delta\rho g}{\rho_s}\right)^{1/2} h_f^{3/2} \end{aligned} \tag{17}$$

For the boundary condition that the flow into the system is a constant Q_0, Q_f equals the inflow rate on the boundary at x=0. From this we can obtain the depth, h_f, and the averaged velocity, $(u_f)_{mean}$, of smoke front(nose).

$$h_f = \left(\frac{3}{2}\right)^{2/3}\left(\frac{2\Delta\rho g}{\rho_s}\right)^{-1/3}\left(\frac{Q_0}{B}\right)^{2/3} \tag{18}$$

$$\begin{aligned} (u_f)_{mean} &= Q_f/(Bh_f) \\ &= 2\cdot3^{-2/3}\left(\frac{\Delta\rho g Q_0}{\rho_s B}\right)^{1/3} \end{aligned} \tag{19}$$

In order to treat the effect of heat loss, we assume that the smoke temperature can be averaged over the length of the corridor, the heat loss to wall and ceiling is treated by:

$$q = \alpha(T_s - T_w) \tag{20}$$

and the temperature of the wall and ceiling are constant in space and time for the early transient conditions.

Under these assumptions, we obtain mass and energy equation for the smoke layer.

Mass conservation of smoke layer can be written as

$$\frac{d(\rho_s V_s)}{dt} = \rho_0 Q_0, \tag{21}$$

and energy conservation as

$$(C_p \rho_s T_s)\frac{dV_s}{dt} = C_p \rho_0 Q_0 T_0 - qA_s, \tag{22}$$

where A_s is the surface area of the wall that the smoke touches. Subscript s refers to the smoke layer and o to the boundary at the inlet.

From eqs (21) and (22), we obtain an equation of the change in the average temperature of the smoke layer,

$$(C_p \rho_s V_s)\frac{dT_s}{dt} = C_p \rho_0 Q_0 (T_0 - T_s) - A_s \alpha (T_s - T_w). \tag{23}$$

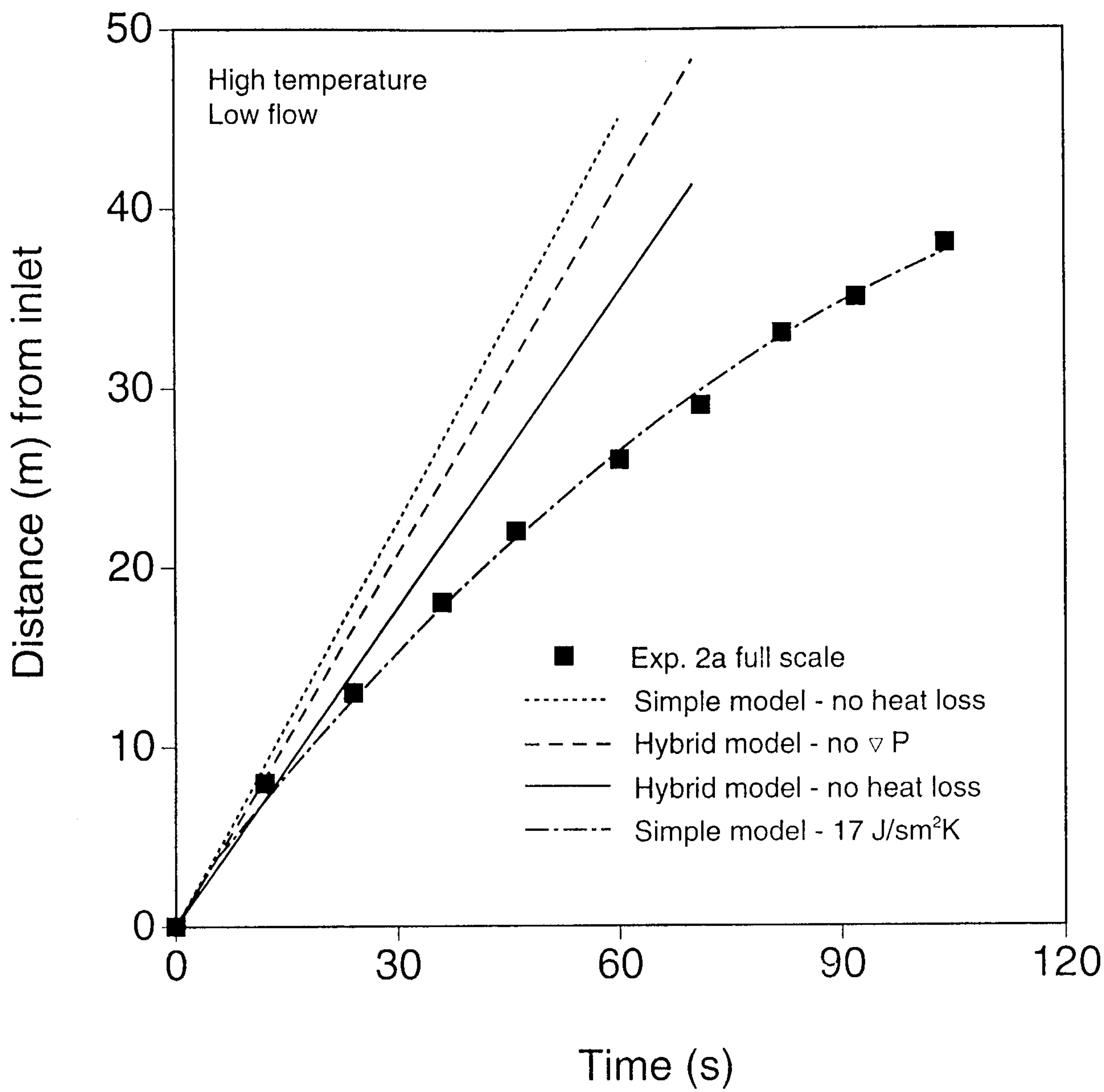
Experiment

We have run a series of experiments to test the validity of the models which we have developed. The experiments of smoke flow in a corridor were done at the Building Research Institute (Japan). Figure 1 shows the plan and cross section of the facility. The length of corridor is about 65 m, the width is 1.5 m and the height of ceiling is 2 m. We used only the first 40 m of the corridor because the width changes to 5 m at that location. To study the effects of a soffit (smoke barrier) on the smoke movement, a soffit is set at 23 m from the smoke inlet. Thermocouples are used for detecting the arrival of the smoke layer. Fifteen experiments were done for the full scale study. Table 1 shows the experimental conditions.

Figure 2 shows the location of the smoke front. This illustrates the drop in the velocity due mostly to heat loss. There is also some effect of the downstream soffit (at 23 m) in constraining smoke movement on both sides of the barrier. However, this effect is not as pronounced as the heat loss mechanism. As the soffit is lowered, the smoke velocity becomes smaller. The depth beyond the soffit is more shallow, and the smoke layer depth on the upstream side of the soffit is deeper. A comparison of all experiments show that the smoke spread is faster for high flow and high temperature, and the effect of heat loss is most important in these cases.

Figure 3 shows the horizontal distribution of the smoke temperature in experiment (5-a). Each point along the horizontal axis represents the maximum temperature on that particular thermocouple tree. Although this also shows the location of the smoke front by measuring the temperature, primarily it illustrates the important point that the maximum temperature is decreasing with distance from the source. This is the effect of heat loss to the surfaces. This effect is not mimicked by the hybrid model, as will be

FIGURE 4


High temperature
Low flow
Exp. 2a full scale
Simple model - no heat loss
Hybrid model - no ▽ P
Hybrid model - no heat loss
Simple model - 17 J/sm²K
Distance (m) from inlet
Time (s)
0
10
20
30
40
50
0
30
60
90
120

seen.

Comparison between the Computer Model and Full Scale Experiment

We compare the results of the full scale experiments with three versions of the model. This elucidates the importance of each effect. Figure 4 shows the comparison between one of the experiments, 2a, and results of the hybrid model and the simple heat loss model.

For the hybrid model, we divided the 40[m] corridor into 200 cells. As discussed earlier, in order for this particular solver to work stably with these equations, we need to include a numerical viscosity term. In this case, we can used a constant ν_{num} of a 0.1 for our system.

Up to about 10 seconds, the hybrid model shows a good agreement for the location of smoke front. After this time, the speed is decreasing in the experiment whereas it is constant in the model. This is due entirely to the fact that there is no heat loss and the gas density for the model is constant for all time. The spread speed of the case when the pressure gradient is ignored is faster than that when it is included. The depth of the smoke layer of the former is less than the latter. These show the effect of the pressure term, and the effect is fairly important.

The system of eqs (19) and (23) gives us the smoke spread speed and the average temperature. For simplicity, we can calculate eq (31) by the explicit method. Figure 4 shows the location of smoke front for the experimental results and the calculation results for heat transfer coefficients 17.4 and 23.3 [W/m^2]. At an early stage the experiment, smoke spread is faster than the predictions of the simple model, but after about 20 seconds the agreement is good for the heat transfer coefficient 17.4.

Conclusions

We have developed a simplified field model to compute the movement of smoke in a corridor. The model is suitable for inclusion in the zone model CFAST. This work has shown that there is reasonably good agreement between the theory (computer model) and experiment even with no heat transfer. The importance of various effects, including heat transfer, has been elucidated. The theory shows a good agreement with the experimental data for a 40[m] full scale corridor up to the point where heat loss becomes significant. The simple model with heat transfer with a constant heat transfer coefficient shows a good agreement with the experiment, but in early time the prediction is smaller than the results of experiment. However, this is not a suitable substitution for the correct calculation, in that we need actual time temperature histories along the corridor to incorporate the effect of sidewall openings, and so on.

Acknowledgement

The authors would like to thank T. Wakamatsu (Tokyo Science University), T. Yamana (BRI), H.Nakamura (Shimizu Corporation), and students of Tokyo Science University for the cooperation of the full scale and small scaled experiments.

Nomenclature

B width of corridor [m]
Cp constant-pressure specific heat [kJ/kg/K]
g gravitational acceleration [m^2/s]
h height of interface or depth of layer [m]
H height of ceiling [m]
L length [m]
P pressure [N/m^2]
q heat transfer rate [kJ/m^2s]
Q flow rate of smoke [m^3/s]
t time [s]
T temperature [K]
u,v,w velocity of x-,y-,z-direction [m/s]
V volume of smoke layer or zone [m^3]
x direction of flow movement
y horizontal diagonal direction of x
z vertical direction
α heat ..ansfer coefficient combined convection and radiation[kW/m^2K]
λ thermal conductivity of wall or ceiling [kW/mK]
ρ density [kg/m^3]
ν_{num} numerical viscosity (cm^2/s)

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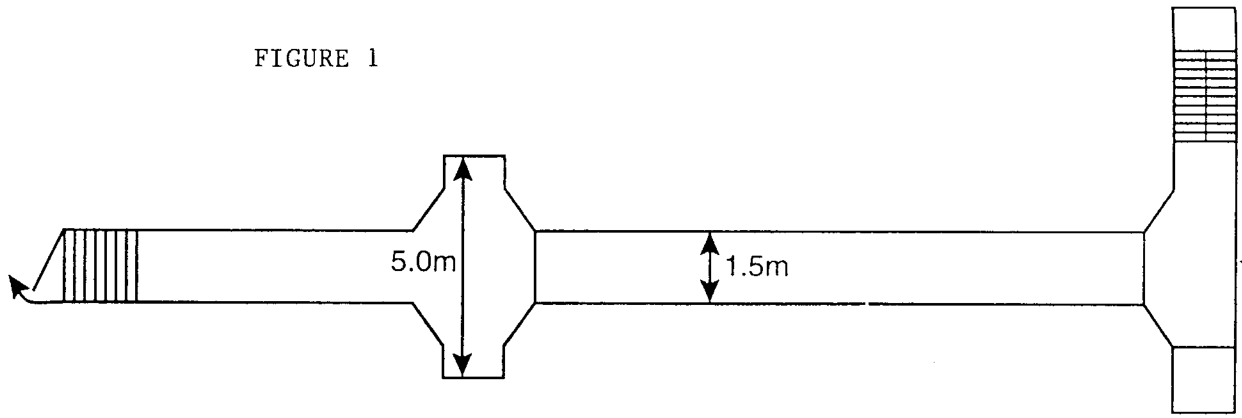
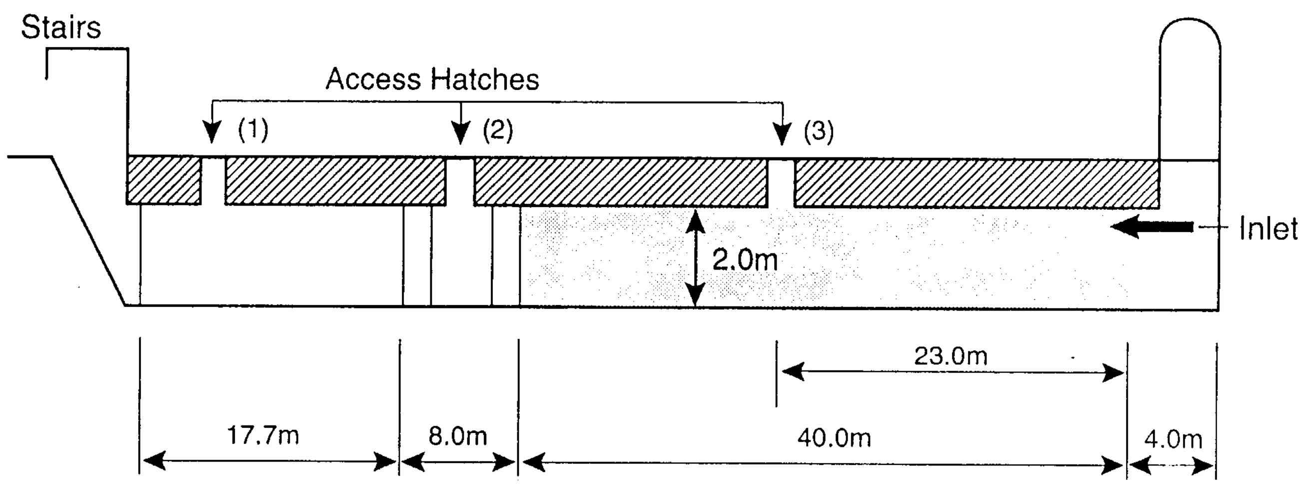
Table 1 Conditions of the Full Scale (Full Scale)

Exp. No.	Soffit Height [m]	Inlet Air Rate [m^3/s]	Inlet Smoke Temp. [°C]	Corridor Temp. [°C]	Outside Temp. [°C]
1-a	0.0	0.248	48.0	23.6	28.0
1-b	0.5	0.257	50.1	24.3	26.5
1-c	1.0	0.246	51.1	24.5	26.6
2-a	0.0	0.233	64.8	23.7	28.2
2-b	0.5	0.238	64.9	24.3	28.0
2-c	1.0	0.245	67.4	23.8	26.6
3-a	0.0	0.401	43.7	24.5	28.1
3-b	0.5	0.402	42.7	24.3	28.1
3-c	1.0	0.375	42.3	22.9	26.3
4-a	0.0	0.398	56.3	23.7	30.0
4-b	0.5	0.383	56.1	24.1	27.5
4-c	1.0	0.381	57.3	23.1	31.8
5-a	0.0	0.360	68.1	22.8	28.4
5-b	0.5	0.393	68.5	24.1	26.6
5-c	1.0	0.395	69.7	24.6	27.3

FIGURE 1



Plan View



Cross Section

Corridor Facility at BRI (Japan)

FIGURE 2

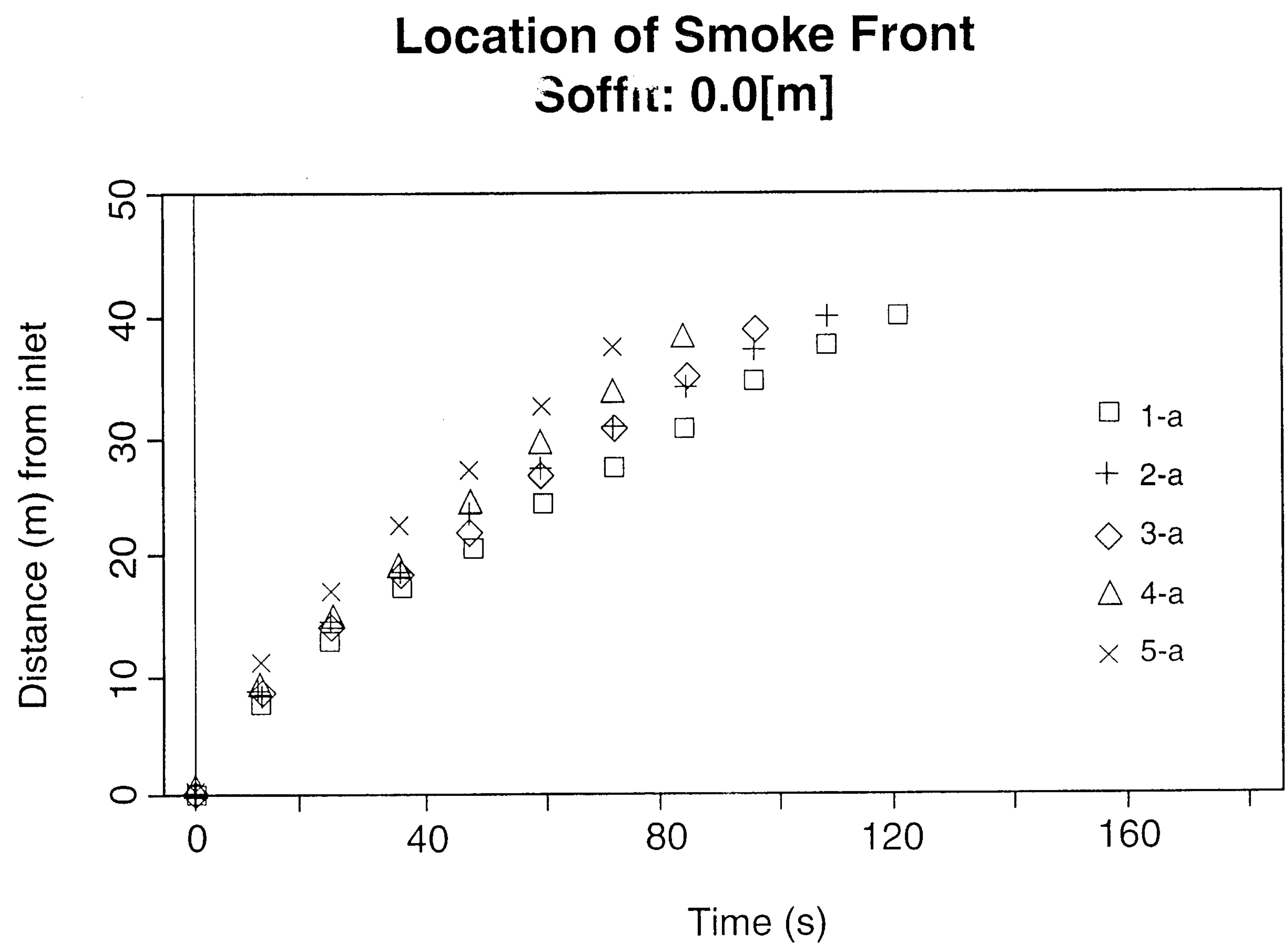
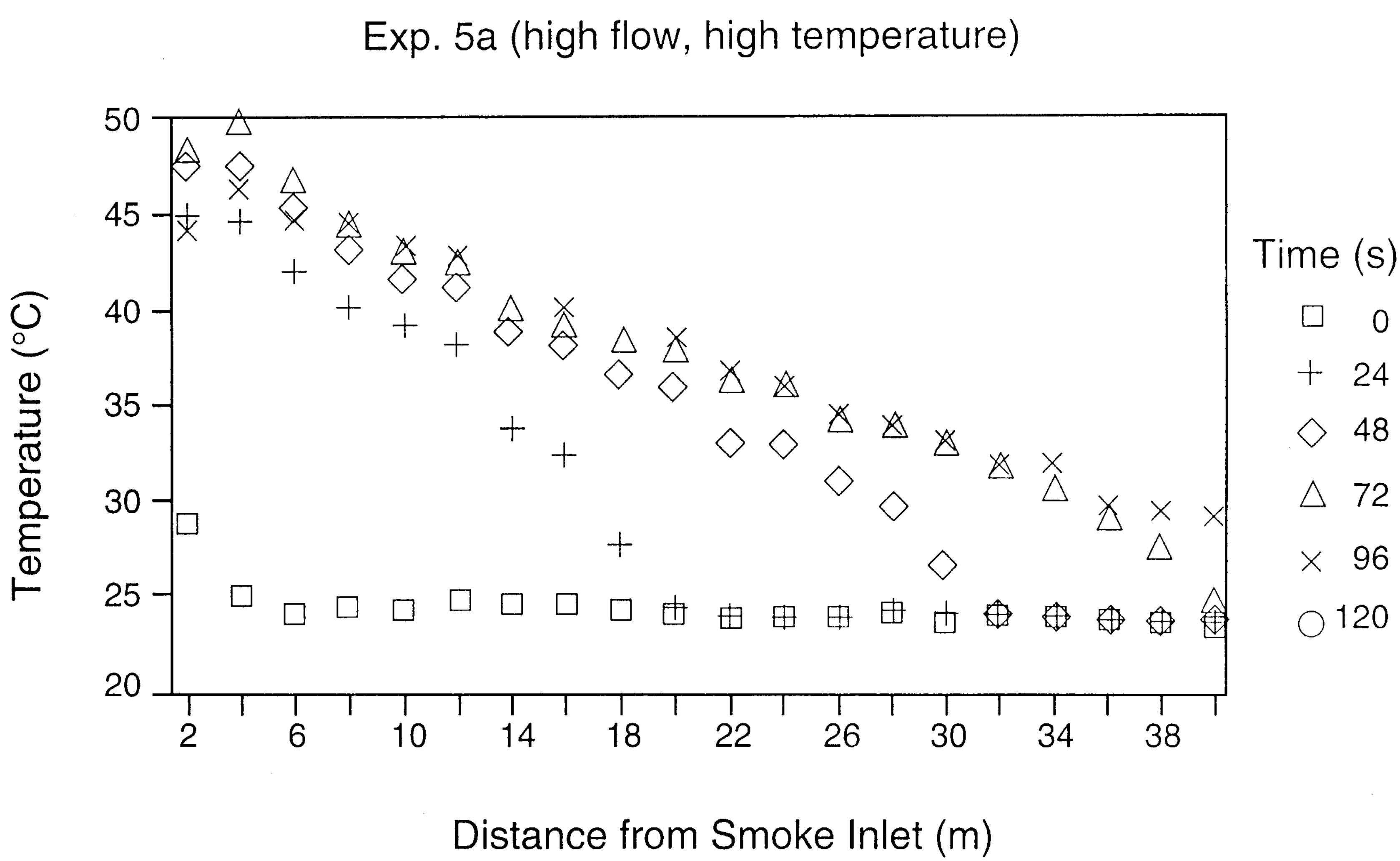

Location of Smoke Front
Soffit: 0.0[m]
Distance (m) from inlet
0
10
20
30
40
50
0
40
80
120
160
Time (s)
1-a
2-a
3-a
4-a
5-a

FIGURE 3


Exp. 5a (high flow, high temperature)
Temperature (°C)
50
45
40
35
30
25
20
2
6
10
14
18
22
26
30
34
38
Distance from Smoke Inlet (m)
Time (s)
0
24
48
72
96
120