RADIATION AND VELOCITY FIELDS INDUCED BY LOCALIZED TEMPERATURE FLUCTUATIONS

by

Howard R. Baum
Building and Fire Research Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899, USA
and
William E. Mell
University of Utah
Department of Chemical and Fuels Engineering
Salt Lake City, UT 84412-1109, USA

Reprinted from the Combustion Institute, Symposium (International) on Combustion, 28th. Proceedings. Volume 1. July 20-August 4, 2000, Edinburgh, Scotland, Combustion Institute, Pittsburgh, PA, Candel, S.; Driscoll, J. F.; Burgess, A. R.; Gore, J. P., Editors, 473-479 pp, 2000.

Proceedings of the Combustion Institute, Volume 28, 2000/pp. 473–479

RADIATION AND VELOCITY FIELDS INDUCED BY LOCALIZED TEMPERATURE FLUCTUATIONS

HOWARD R. BAUM[1] AND WILLIAM E. MELL[2]

[1]*National Institute of Standards and Technology*
Building and Fire Research Laboratory
Gaithersburg, MD 20899-8640 USA
[2]*The University of Utah*
Department of Chemical and Fuels Engineering
Salt Lake City, Utah 84412-1109 USA

A theory describing the coupling between radiative transport, thermal conduction, and velocity fluctuations in postcombustion gases is described. The emission and absorption of radiant energy are taken to be dominated by soot, distributed uniformly in space. The theory is local in the sense that the gas is assumed to be unbounded. However, the temperature, velocity, and radiation fields can be both three-dimensional and time-dependent. Moreover, the model can be thought of as describing any postcombustion scenario in which the absorption coefficient is spatially uniform. Within the framework of the low Mach number combustion equations, an exact representation of the velocity and radiation fields instantaneously induced by fluctuations of any magnitude in the temperature is presented. This result is used to derive a single scalar integro-differential equation for the temperature that incorporates the conservation of mass and energy, together with an exact solution of the radiative transport equation. Some consequences of the theory are illustrated by studying the response generated by a spherically burning fuel mass.

Introduction

The products of combustion resulting from the burning of most hydrocarbon fuels are strong emitters and absorbers of radiant energy. This is particularly true of large fire scenarios, where soot particulate can account for 10%–15% of all the fuel mass. Thus, the coupling between radiation fields and the convective transport of mass and energy is a subject of considerable importance. Two conditions must be satisfied for these effects to be significant. First, the fluctuations in temperature must be large enough for radiative losses from high-temperature regions to be competitive with thermal conduction. This implies that the Boltzmann number, $\mathcal{B}$, defined as a ratio of conduction to radiation energy flux (see equation 21) must be small. Second, the spatial domain occupied by the combustion products must be large enough for the radiation to be reabsorbed within the region of interest. Thus, if κ is the absorption coefficient for the medium, and L is a macroscopic length scale, then $\kappa L \sim O(1)$.

If the first of these conditions is not satisfied, then radiative transport is unimportant. This is the case typically treated in most direct numerical simulations of turbulent combustion [1]. If $\kappa L \ll 1$, then the emitted radiation is lost to the boundaries of the domain. In this case, which is common in enclosure fire scenarios, the radiative losses are quite important, but radiative transport is not coupled to the flow field [2]. Two-dimensional transient simulations of reacting flows with radiation losses have been performed [3]. There also exist many simulations of both enclosure fires [4] and outdoor scenarios [5] that couple the radiative transport to the convective transport and combustion models. However, all such calculations rely on turbulence models that ignore any interaction between temperature fluctuations and the radiation field. Moreover, there is no way to study the relationship between the velocity fluctuations that are inevitably introduced by the temperature and radiation fluctuations that result from the combustion processes.

This paper describes a theory of coupled radiative transport and velocity fluctuations induced by local temperature fluctuations in postcombustion gases. The radiative properties are assumed to be dominated by soot, which is uniformly distributed in space. The theory is local in the sense that the gas is taken to be unbounded, with a spatially uniform absorption coefficient. However, the temperature can vary spatially in three dimensions, subject only to the condition that the fluctuations are confined to a finite domain. The gray gas approximation is made, with the Planck mean absorption coefficient given by Atreya and Aggrawal as $\kappa = 11.86 f_v T$ [6]. This representation of the absorption coefficient shows that the soot mass fraction rather than the volume fraction must be taken as constant for consistency. An extended soot cloud with internal temperature fluctuations is certainly a plausible scenario for large fire problems. Moreover, since the resulting theory is cast in terms of exact analytical relationships, given the constant coefficient gray gas assumption, it may prove useful in other applications. Specifically, the ability to couple radiative transport to a three-dimensional transient simulation with little computational overhead may be an attractive prospect for direct numerical simulations of turbulent combustion.

The goal of the analysis is to determine the radiative heat-flux distribution and associated velocity field as a function of the temperature fluctuation. This is accomplished by employing a decomposition of the velocity field into a solenoidal component containing the vorticity and an irrotational component accounting for the thermal effects. This has proved an effective tool for analyzing fire-induced flow fields both theoretically [7] and experimentally [8]. The divergence of the radiative heat flux interacts instantaneously with the gas to produce the local expansion. Under the assumptions stated above, the radiative heat-flux vector is itself the gradient of a scalar potential. This irrotational expansion is also affected instantaneously by both the temperature field and any chemical heat release occurring in the gas. Thus, the radiation, temperature, and velocity fields are closely correlated. The relationships between these quantities can be established explicitly given the assumptions outlined above. In the next section the mathematical model is developed as a three-dimensional time-dependent theory. Following this, the special case of a spherically symmetric fluctuation field is considered in detail.

Mathematical Model

Consider a non-uniformly heated gas occupying an unbounded domain. The gas is assumed to be a gray radiatively absorbing medium with spatially homogeneous optical properties. The temperature field in the gas has localized "hot spots" which may be of any magnitude or shape but ultimately decay to a uniform ambient temperature T_∞. Let κ be the absorption coefficient and $I(\vec{r}, \vec{\Omega})$ be the radiant intensity. Since the gas is at ambient temperature far from all the heated regions, it is convenient to write the radiant intensity in the form

$$I(\vec{r}, \vec{\Omega}) = \frac{\sigma T_\infty^4}{\pi} + J(\vec{r}, \vec{\Omega}) \tag{1}$$

Then, $J(\vec{r}, \vec{\Omega})$ satisfies the equation

$$\vec{\Omega} \cdot \nabla J(\vec{r},\vec{\Omega}) + \kappa J(\vec{r},\vec{\Omega}) = \frac{\kappa\sigma}{\pi}[T(\vec{r})^4 - T_\infty^4] \tag{2}$$

The only boundary condition required is that J vanish everywhere far from all the heated regions. The quantity of most physical interest is the radiative heat-flux vector $\vec{q}(\vec{r})$. It is defined in terms of J through the relation

$$\vec{q}(\vec{r}) = \int \vec{\Omega} J(\vec{r}, \vec{\Omega})\, d\vec{\Omega} \tag{3}$$

The divergence of $\vec{q}$ is the net radiant energy flux emitted from any point in the gas. This follows directly from the integral of equation 2 over all $\vec{\Omega}$. The result is the radiant energy balance

$$\nabla \cdot \vec{q}(\vec{r}) = 4\kappa\sigma[T(\vec{r})^4 - T_\infty^4] - \kappa \int J(\vec{r}, \vec{\Omega})\, d\vec{\Omega} \tag{4}$$

The net energy emitted by radiation represents a local sink of energy in the gas. As such, it induces temperature and velocity fields whose spatial distribution is determined by the conservation laws of fluid mechanics. The equations expressing the conservation of mass and energy in the low Mach number approximation to reacting gas dynamics [9] take the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{5}$$

$$\rho \left(\frac{\partial h}{\partial t} + \vec{u} \cdot \nabla h \right) = \nabla \cdot (k \nabla T) - \nabla \cdot \vec{q} + Q_c \tag{6}$$

Here, ρ, $\vec{u}$, and h are, respectively, the density, velocity, and sensible enthalpy in the gas. The quantity k is the thermal conductivity, while Q_c represents the rate of heat release per unit volume liberated by chemical reactions.

For reasons that will become clearer below, the momentum conservation equations are not needed for the present analysis. However, the conservation laws must be supplemented by an equation of state which can be written in the form

$$\rho h = \rho_\infty h_\infty \tag{7}$$

Here, the subscript ∞ refers to ambient conditions far from the heated regions of the domain. Physically, equation 7 assumes that the pressure will not change much from its ambient value if the Mach number remains small. In addition, the number of internal degrees of freedom excited at the temperature range under consideration is taken to be the same for each of the gaseous species that make up the fluid.

The velocity field can always be decomposed into a solenoidal divergence-free component $\vec{v}$ and an irrotational field $\nabla\phi$. If the energy equation is added to h times the mass conservation equation, the result is an equation for the velocity potential in the form

$$\rho_\infty h_\infty \nabla^2 \phi = \nabla^2 \left(\int_{T_\infty}^{T} k(T)\, dT \right) - \nabla \cdot \vec{q} + Q_c \quad (8)$$

The radiant heat flux can itself be expressed as the gradient of a potential Ψ when the absorption coefficient is spatially uniform. This representation is derived explicitly for the case of a purely absorbing medium [10]. Moreover, when scattering is included, a similar representation (to be presented elsewhere) can be derived. Thus, the radiant heat flux can be written as

$$\vec{q} = -\frac{1}{\kappa} \nabla \Psi \quad (9)$$

$$\Psi = \int \frac{\kappa\sigma}{\pi} (T^4(\vec{r}_o) - T_\infty^4) \frac{\kappa^2}{x} E_2(x)\, d^3 r_o \quad (10)$$

$$x = \kappa |\vec{r} - \vec{r}_o| \quad (11)$$

Here, $E_2(x)$ denotes the exponential integral function, and the integral is taken over all space. Substitution of equation 9 into equation 8 and use of the requirement that all potentials vanish far from the heated regions yields the following general formula for the velocity potential ϕ.

$$\phi = \frac{1}{\rho_\infty h_\infty} \left(\int_{T_\infty}^{T} k(T')\, dT' + \frac{\Psi}{\kappa} \right) + \phi_c \quad (12)$$

$$\nabla^2 \phi_c = \frac{Q_c}{\rho_\infty h_\infty} \quad (13)$$

The above result has a simple physical interpretation. The velocity potential consists of two basic parts. First, there is a local volume flux exactly opposed to the two components of the heat flux induced by thermal conduction and radiative transport, respectively. This velocity supplies exactly the amount of mass needed to ensure mass conservation for given local fluctuations in temperature. Note that this flow is a redistribution rather than a creation of volume. Second, a distributed source is generated by the heat released by combustion, Q_c. This component of the velocity is a true source of volume. It only acts while combustion energy is being released, in contrast to the other components of the potential field. However, while this term is active, it represents a long range rather than a local contribution to the velocity.

The above results can now be combined into a generalized conservation law that incorporates convective, diffusive, and radiative transport of mass and energy into a single equation. This is accomplished by substituting the various components of the velocity field into the mass conservation equation 5 to obtain

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho(\vec{v} + \nabla \phi_c)) + \frac{1}{\rho_\infty h_\infty} \nabla \cdot (\rho k \nabla T) = -\frac{1}{\rho_\infty h_\infty \kappa} \nabla \cdot (\rho \nabla \Psi) \quad (14)$$

The terms on the left-hand side of equation 14 represent, in turn, the contributions of the unsteadiness, mixing, expansion, and diffusion to the mass balance, while the right-hand side represents the redistribution induced by radiative transport. All terms except for the solenoidal velocity $\vec{v}$ and heat release are defined in terms of the temperature. Thus, once a combustion model has been chosen, the only other equations needed to close the system are the momentum equations and the solenoidal condition given by

$$\nabla \times \vec{v} = \vec{\omega}; \qquad \nabla \cdot \vec{v} = 0 \quad (15)$$

The momentum equations in effect determine the vorticity $\vec{\omega}$, while the solenoidal condition determines the corresponding velocity field.

Spherical Temperature Fluctuation

The processes described above can be best illustrated by considering a simple example. Assume that a large region with a uniform soot mass fraction and temperature at rest is subjected to a spherically symmetric temperature fluctuation. The spherical symmetry guarantees that the resulting motion is a potential flow. Attention is focused on the postcombustion decay of the temperature field under the combined influence of radiation and diffusion. The initial temperature field is determined from a simple mixture-fraction-based combustion model that does not explicitly deal with radiative transport [11]. More detailed solutions to this problem including soot formation and radiative losses have been obtained [12,13]. However, the approach used by these authors is quite different from that employed here, and the full radiative transport equation is not solved.

To proceed, equation 12 is substituted into the mass conservation equation, which takes the following form for a spherically symmetric problem in the absence of combustion

$$\frac{\partial \rho}{\partial t} + \frac{1}{\rho_\infty h_\infty} \frac{1}{r^2} \frac{\partial}{\partial r} \left(\rho r^2 \left[k(T) \frac{\partial T}{\partial r} + \frac{1}{\kappa} \frac{\partial \Psi}{\partial r} \right] \right) = 0 \quad (16)$$

The next step is the calculation of the radiative contribution to equation 16. To take advantage of the symmetry in Ψ, it is convenient to introduce a spherical coordinate system in the $\vec{r}_o$ space with $\vec{r}$ as the pole and θ_o as the polar coordinate. Then

$$x = \kappa \sqrt{r^2 + r_o^2 - 2 r r_o \cos \theta_o} \quad (17)$$

Since T depends only on r_o, the integrand in equation 10 is independent of the azimuthal coordinate.

The integration over θ_o can then be carried out using $x(\cos\theta_o)$ as the variable of integration. The result is

$$-\frac{1}{\kappa}\frac{\partial\Psi}{\partial r} = \frac{1}{r^2}\int_0^\infty dr_o r_o^2\, \kappa\sigma\, (T^4(r_o) - T_\infty^4)$$

$$(\tilde{I}(\kappa|r - r_o|) - \tilde{I}(\kappa(r + r_o))) \tag{18}$$

$$\tilde{I}(z) = \frac{1}{\kappa r_o}\left(\exp(-z) + \kappa^2(r^2 - r_o^2)\frac{E_2(z)}{z}\right) \tag{19}$$

The problem is made non-dimensional as follows: the temperature is represented in the form $T = T_\infty\Theta(y,\ \tau)$. The dimensionless radial coordinate y and time τ are given by

$$y = \kappa r,\ \tau = \frac{\kappa^2 k_\infty}{\rho_\infty C_p}\, t \tag{20}$$

The ratio of conduction energy flux to radiatively induced energy flux is characterized by the Boltzmann number, $\mathcal{B}$, defined here as

$$\mathcal{B} = \frac{\kappa k_\infty T_\infty}{\sigma T_\infty^4} \tag{21}$$

Then, assuming the specific heat C_p and the product ρk are not functions of temperature, equation 16 takes the form

$$\frac{1}{\Theta^2}\frac{\partial\Theta}{\partial\tau} + \frac{1}{\mathcal{B}}\frac{1}{y^2}\frac{\partial}{\partial y}\left(\frac{1}{\Theta}\int_0^\infty dy_o\,[\Theta^4(y_o) - 1]\ y_o K(y, y_o)\right) = \frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial\Theta}{\partial y}\right) \tag{22}$$

$$K(y, y_o) = \exp(-|y - y_o|) - \exp(-(y + y_o)) + (y^2 - y_o^2)\left[\frac{E_2(|y - y_o|)}{|y - y_o|} - \frac{E_2(y + y_o)}{(y + y_o)}\right] \tag{23}$$

Equation 22 is the simplest multidimensional model that incorporates all the conservation laws for both matter and radiation in a single equation. It is multidimensional not only in the sense that it incorporates spherical symmetry as opposed to a planar geometry, but it also allows many replicates of this spherical object to coexist in the same region of interest. This coexistence is possible because the influence of each heated sphere decays exponentially with distance from its center. Thus, if the centers of each of many such objects are spaced several times the absorption length, κ^{-1}, apart, their interaction is negligible. Under these circumstances, a library of spherical radiating objects of varying size and temperature can become the basis for an averaging process that would assess the effects of spatial fluctuations in temperature on emitted radiant heat flux.

The initial condition is determined by developing an approximate solution for the combustion of a spherical fuel blob in an oxidizing environment at rest. The soot (or other absorbing material) in the environment is assumed to be a product of combustion generated elsewhere in a larger domain, which is not the object of the current study. The initial radius R_o of the blob is small in the sense that $\kappa R_o \ll 1$. It is further assumed, following Ref. [11], that the specific volume is a piecewise linear function of the mixture fraction $Z(r, t)$. Under these circumstances, the mass conservation and mixture fraction equations form the basis of the solution. If r is the local spherical radial coordinate measured from the center of the blob, and v_r is the radial velocity, these equations take the form

$$\frac{\partial}{\partial t}(\rho r^2) + \frac{\partial}{\partial r}(\rho v_r r^2) = 0 \tag{24}$$

$$\rho\left(\frac{\partial Z}{\partial t} + v_r\frac{\partial Z}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho D r^2\frac{\partial Z}{\partial r}\right) \tag{25}$$

The equations are solved by introducing a space-like Lagrangian variable $s(r, t)$ defined as follows

$$\frac{\partial}{\partial r}(\rho_\infty s^3) = 3\rho r^3 \quad \frac{\partial}{\partial t}(\rho_\infty s^3) = -3\rho v_r r^3 \tag{26}$$

Then, using $(s,\ t)$ as the independent variables and defining the variable diffusivity D by the relation $\rho^2 D = \rho_\infty^2 D_\infty$, the mixture fraction equation becomes

$$\frac{\partial Z}{\partial t} = \frac{D_\infty}{s^2}\frac{\partial}{\partial s}\left(\frac{r^4}{s^2}\frac{\partial Z}{\partial s}\right) \tag{27}$$

The subscript ∞ denotes ambient values of all physical quantities. Up to this point, equation 27 is exact, given the assumptions stated above. An approximate analytical solution is obtained by replacing the physical radius r by s in the right-hand side of the equation. This choice vanishes linearly at the origin as it should, and is correct asymptotically as $r \to \infty$. Since $r > s$ due to the expansion of the gas, the effect of the approximation is to lessen the temperature dependence of the diffusivity, which is realistic. The result is the spherical diffusion equation, which is solved subject to the initial condition

$$Z(s,\ 0) = H((\rho_o/\rho_\infty)^{1/3}\, R_o - s) \tag{28}$$

Here, H denotes the Heaviside step function, and ρ_o is the initial fuel density. The solution for Z takes the form

$$Z = \frac{1}{2}\left(\operatorname{erf}\left[\frac{1-x}{2\sqrt{\tau}}\right] + \operatorname{erf}\left[\frac{1+x}{2\sqrt{\tau}}\right] + \frac{2}{x}\sqrt{\frac{\tau}{\pi}}\left[\exp\left(-\left[\frac{1+x}{2\sqrt{\tau}}\right]^2\right) - \exp\left(-\left[\frac{1-x}{2\sqrt{\tau}}\right]^2\right)\right]\right) \tag{29}$$

$$s = (\rho_o/\rho_\infty)^{1/3}\, R_o x, \quad t = (\rho_o^*/\rho_\infty)^{2/3}\,\frac{R_o^2}{D_\infty}\,\tau \tag{30}$$

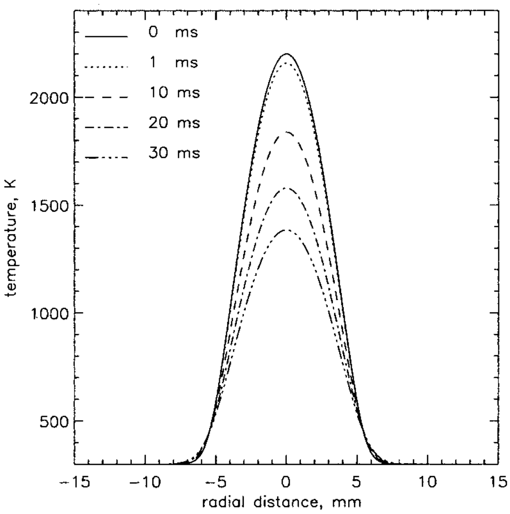

FIG. 1. Time evolution of radial profiles of temperature in the case without radiation (as computed from equations 29–32). Five different times are shown. The $t = 0$ ms curve was used as the initial condition for the computation with radiation (equation 22).

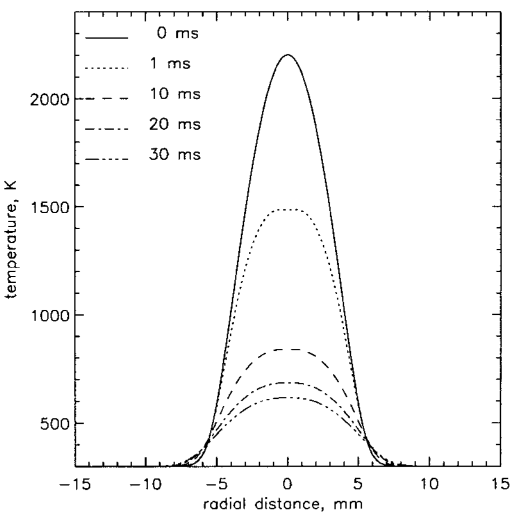

FIG. 2. Temperature profiles for the $\mathcal{B} = 0.5$ case.

The physical coordinate is recovered by integrating the first of equations 26 and using the postulated linear relation between specific volume and mixture fraction. This procedure guarantees that mass is exactly conserved in the solution. The physical quantity of actual interest is the radial temperature profile at the instant of burnout. At burnout, the flame sheet value Z_f and corresponding density ρ_f have moved to the point $r = s = 0$. For all later times, $Z \leq Z_f$ everywhere. Thus, the relation between $r = (\rho_o/\rho_\infty)^{1/3} R_o \tilde{r}$ and the (s, t) coordinate system becomes

$$\tilde{r}^3 = x^3 + \frac{3}{Z_f}\left(\frac{\rho_\infty}{\rho_f} - 1\right)(F(x, \tau) - F(-x, \tau)) \tag{31}$$

$$F(x,\tau) = \frac{1}{6}\left((x^3 + 1)\,\mathrm{erf}\left[\frac{1+x}{2\sqrt{\tau}}\right] + \frac{2}{3}\sqrt{\frac{\tau}{\pi}}\exp\left(-\left[\frac{1+x}{2\sqrt{\tau}}\right]^2\right)(1 - 2\tau + x(x-1))\right) \tag{32}$$

Equations 29–32 implicitly define the solution for the mixture fraction. The major species are also linearly related to the mixture fraction. Use of these relations together with the equation of state then yields the temperature distribution. The dimensionless burnout time τ_b is the unique solution of the equation $Z(0, \tau_b) = Z_f$. Substituting this value into the above equations and state relations then yields the combustion-generated starting temperature profile.

Results and Discussion

Calculations were performed using acetylene as the fuel in a 21% O_2/79% N_2 oxidizing environment generating H_2O and CO_2 in stoichiometric proportions. While fires do not typically yield combustion products in stoichiometric proportions, the temperature fields generated by the calculation are representative enough to illustrate the theory. An explicit, variable time step, second-order, Runge-Kutta method was used to time-step the integro-differential equation 22. An approach based on Romberg's method was used to compute the integral in equation 22. The kernel, equation 23, is not singular but does have a discontinuity at $y = y_o$. Results using increasingly fine grids were compared to ensure that there was no grid dependency.

All results shown here are for a spherical fuel blob with initial radius $R_o = 5$ mm and $\kappa R_o \ll 1$. As discussed above, the initial condition was obtained from a mixture-fraction-based model for the combustion of a spherical fuel blob, equations 29–32, which did not include radiative transfer. Choosing κ such that $\kappa R_o \ll 1$ ensured that the combustion burnout time was much shorter than the characteristic timescale in the radiation fluctuation model. Fig. 1 is a plot of the time evolution of temperature profiles as obtained from the mixture fraction-based model. The solid curve represents the temperature at burnout and was used for the initial condition in equation 22.

Figures 2 and 3 show the evolution of temperature profiles for two Boltzmann cases (i.e., two κ cases): $\mathcal{B} = 0.5$ and 0.05, respectively. Times are identical

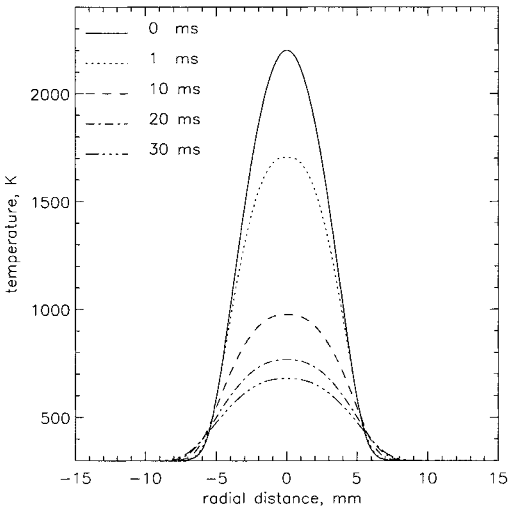

FIG. 3. Temperature profiles for the $\mathcal{B} = 0.05$ case.

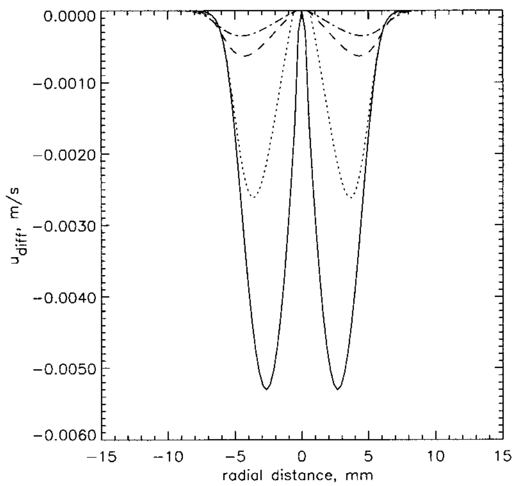

FIG. 5. Profiles of the diffusion-induced velocity, $\mathcal{B} = 0.5$.

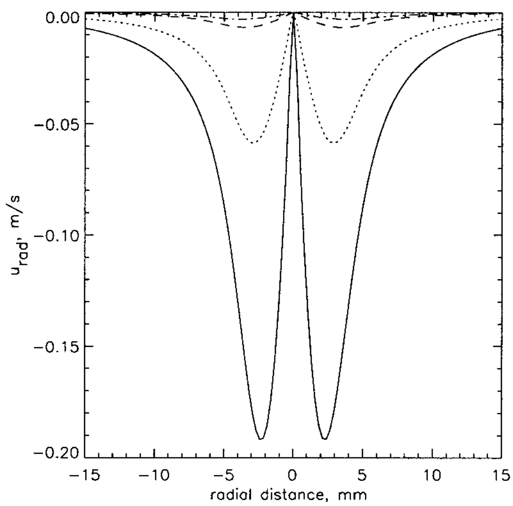

FIG. 4. Profiles of the radiation-induced velocity, $\mathcal{B} = 0.5$.

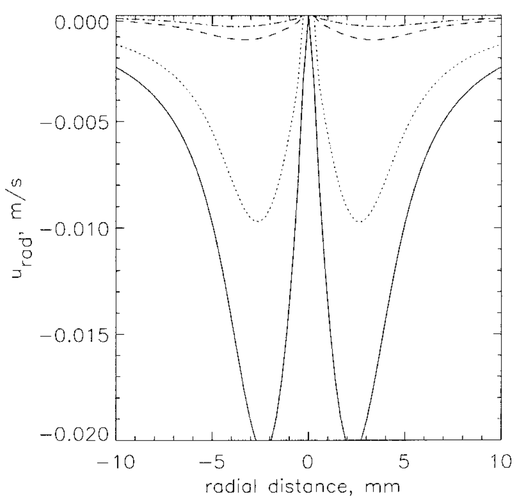

FIG. 6. Profiles of the radiation-induced velocity, $\mathcal{B} = 0.05$.

to those in Fig. 1. Both Fig. 2 and Fig. 3 show that radiation energy transfer significantly increased the temperature decay (compare to Fig. 1). The decay was most rapid initially, while the temperature was relatively high.

Radial profiles of the radiation- (u_{rad}) and diffusion- (u_{diff}) induced components of the velocity, $\nabla\phi$ (see equation 13), are plotted in Figs. 4 and 5, respectively, for the $\mathcal{B} = 0.5$ case. The profiles are at the same times as in the previous figures. It is clear that the radiative flux was the dominant contributor to the velocity. For example, at the earliest time (solid line), the smallest value of u_{rad} at $r = \pm 15$ mm was larger than the maximum value of u_{diff}. The effect of radiation absorption can also be seen: the profiles for u_{diff} are much more confined radially. For this Boltzmann case (e.g., heavy soot loading), it was essential that the numerical computation include radiative heat transfer to accurately simulate the velocity behavior.

In the $\mathcal{B} = 0.05$ case (see Fig. 6), u_{rad} was also larger than u_{diff} but was smaller than u_{rad} in the $\mathcal{B} = 0.5$ case. This may seem contrary to the interpretation of $\mathcal{B}$ as the ratio of conduction energy flux over radiatively induced energy flux. It should be noted that $\mathcal{B}$(see equation 22) is the ratio of the divergence of the fluxes. In the $\mathcal{B} = 0.5$ case, the length over which radiative absorption occurs was much smaller than the $\mathcal{B} = 0.05$ case. This resulted in a relatively large radiative flux (and therefore radiatively induced velocity) over length scales at which diffusion was relevant.

Conclusions

A theory describing the coupling between radiative transport, thermal conduction, and velocity fluctuations in postcombustion gases was presented. The emission and absorption of radiant energy were taken to be dominated by soot, distributed uniformly in space. The model can be thought of as describing any postcombustion scenario in which the absorption coefficient is spatially uniform. Within the framework of the low Mach number combustion equations, an exact representation of the velocity and radiation fields instantaneously induced by fluctuations of any magnitude in the temperature was developed. This result was used to derive a single scalar integro-differential equation for the temperature that incorporates the conservation of mass, energy, and radiation. Some consequences of the theory were illustrated by studying the response generated by a spherically burning fuel mass. For example, the dominance of radiation over diffusion, in terms of determining the magnitude and spatial range of velocity fluctuations, was clearly seen.

REFERENCES

1. Grinstein, F. F., and Kailsanath, K., *Proc. Combust. Inst.* 26:91–96 (1996).
2. Wen, J. S., Huang, L. Y., Amin, E. M., and Nolan, P., *Proc. Combust. Inst.* 27:2881–2886 (1998).
3. Patnaik, G., and Kailsanath, K., *Proc. Combust. Inst.* 27:711–717 (1998).
4. Lewis, M. J., Moss, J. B., and Rubini, P. A., in *Fire Safety Science, Proceedings of the Second International Symposium* (Y. Hasemi, ed), International Association for Fire Safety Science, Boston, MA, 1997, pp. 463–474.
5. Makhviladze, G. M., Roberts, J. P., and Yakush, S. E., *Combust. Flame* 118:583–605 (1999).
6. Atreya, A., and Agrawal, A., *Combust. Flame* 115:372–382 (1998).
7. Baum, H. R., and McCaffrey, B. J., in *Fire Safety Science, Proceedings of the Second International Symposium,* Hemisphere, New York, 1988, pp. 129–148.
8. Zhou, X. C., Gore, J. P., and Baum, H. R., *Proc. Combust. Inst.* 26:1453–1459 (1996).
9. Rehm, R. G., and Baum, H. R., *J. Res. Natl. Bur. Stand.* 83:297–308 (1978).
10. Baum, H. R., and Mell, W. E., *Combust. Theory Model.* 2:405–422 (1998).
11. Baum, H. R., Rehm, R. G., and Gore, J. P., *Proc. Combust. Inst.* 23:715–722 (1990).
12. Ezekoye, O. A., and Zhang, Z., *Combust. Sci. Technol.* 106:363–382 (1995).
13. Zhang, Z., and Ezekoye, O. A., *Combust. Sci. Technol.* 112:231–255 (1996).

COMMENTS

Carlos Fernadez-Pello, University of California—Berkeley, USA. Why do the thermal effects appear in the potential field and not in the solenoidal field?

Author's Reply. In general, temperature fluctuations affect the solenoidal field as well as the potential field. However, the time scale for a significant response is very different for the two components of the velocity field. A local temperature gradient changes the convective derivative of the circulation about a material fluid element. Thus, the temperature gradient must persist for some time before the local vorticity can build up enough to affect the solenoidal field. The potential field, by contrast, generates a finite response instantaneously according to the low Mach number combustion equations. In reality, this means that the time scale for the response to be effective is the passage time for a sound wave across the material element. In applications where the low Mach number equations are valid, such as most fire scenarios, the potential field will change much more rapidly than the solenoidal field under the influence of a given temperature fluctuation. The response is also more localized spatially, as it is confined by the absorption length scale in the vicinity of the fluctuation. The spherical fluctuation was chosen for the numerical example used to illustrate the theory because in a spherically symmetric geometry, vorticity is kinematically impossible.

•

Jay Gore, Purdue University, USA. Thank you for depicting an important effect. In my opinion, this effect seldom appears in the combustion literature because of its masking by the uncertainties in the volumetric heat-release-rate models. Inclusion of the volumetric radiation sink/source term will in fact help better calibration of the heat-release-rate models. Therefore, this is a very important contribution.

Author's Reply. Thanks for your support. It would be very interesting to devise an experiment that can measure this effect directly.