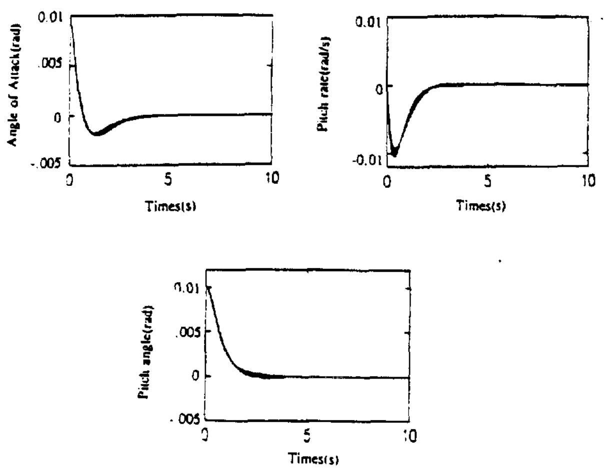

Fig. 2 Responses of uncertain systems with respect to 32 vertices of the uncertain parameter region

That is, for any $0 < 1 < \beta$ and positive semidefinite matrix S,

$$W(\beta q_1 + (1 - \beta)q_2, S) \leq \beta W(q_1, S) + (1 - \beta)W(q_2, S)$$

Therefore $W(q,.)$ is also convex. The maximum is only achieved on the corners of the polytope. Hence the maximization in (9) is reduced to comparing the function values on the corners.

The method developed in the above is applied in design of the longitudinal control of a small aircraft, the Trinidad 20. The linearized longitudinal motion model is given by Mora-Camino and Chaibou (1993). The uncertain parameters are aerodynamic derivatives whose values are dependent on the current point in the flight envelope and evolve the limits

$$-1.63 \leq Z_\alpha \leq -1.41,$$

$$0.09 \leq Z_\delta \leq 0.104,$$

$$-8.69 \leq M_\alpha \leq -7.52,$$

$$-1.94 \leq M_q \leq -1.68,$$

$$-9.34 \leq M_\delta \leq -8.09.$$

The open-loop system is unstable. We want to assign all poles of the uncertain systems within the disk $D(-4, 3.5)$.

Using the design method developed in this technical brief, we obtain

$$\mu = -0.5184 < 0$$

and the state feedback gain for robust pole placement is given by

$$K = [-0.9434 \quad 0.3964 \quad 0.8043]$$

Following Theorem in the above, all poles of the closed-loop systems under the uncertain parameters are within the desired region D(−4, 3.5).

Suppose the initial angle of attack and the pitch angle errors are 0.01 rad. The responses of the uncertain systems with respect to the 32 corners of the uncertain parameter region are shown in Fig. 2. As seen from Fig. 2, the variations in the pitch angle and the angle of attack responses are very small. The pitch rate response has a little bit large variation but it is also very small. Hence the resultant closed-loop systems have nice performance robustness against the parameter variations.

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Melnikov-Based Open-Loop Control of Escape for a Class of Nonlinear Systems

Emil Simiu [1] **and Marek Franaszek** [1]

The performance of certain nonlinear stochastic systems is deemed acceptable if, during a specified time interval, the systems have sufficiently low probabilities of escape from a preferred region of phase space. We propose an open-loop control method for reducing these probabilities. The method is applicable to stochastic systems whose dissipation- and excitation-free counterparts have homoclinic or heteroclinic orbits. The Melnikov relative scale factors are system properties containing information on the frequencies of the random forcing spectral components that are most effective in inducing escapes. Numerical simulations show that substantial advantages can be achieved in some cases by designing control systems that take into account the information contained in the Melnikov scale factors.

Introduction

In this work we discuss a Melnikov-based procedure aimed at achieving efficient stabilization by open-loop control. The proposed procedure is applicable to the class of multistable systems with stochastic excitation, whose dissipation- and forcing-free counterparts possess homoclinic or heteroclinic manifolds. Examples are the motion of a ship subjected to wave loading, as modeled by a second-order equation of motion with a nonlinear restoring term (Hsieh et al., 1994), the rf-driven Josephson junction, and higher- or infinitely-dimensional systems such as buckled columns (Franaszek and Simiu, 1996), mechanical devices (Wiggins and Shaw, 1988), and flows over a corrugated surface (Allen et al., 1991, Simiu, 1996). We review in the following section the theoretical basis of our procedure. Next, we test its effectiveness by using numerical simulations.

Melnikov Processes and Exits From a Well

For a class of dynamical systems described later in this section, the Melnikov approach is a technique providing necessary conditions for the occurrence of chaos—and of exits from preferred (or "safe") regions of phases space. Originally the Melnikov approach was applied only to deterministic systems, including systems with quasiperiodic excitation (Beigie et al., 1991). However, the result was recently extended to systems with stochastic excitation (Frey and Simiu, 1993). One remarkable result of this extension is that, under certain conditions, a motion can be both stochastic *and* chaotic (i.e., sensitive to

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initial conditions). (See also Seki et al., 1993.) For a brief review of stochastic Melnikov theory, see Sivathanu et al. (1995). We note that the theory is useful in applications even if the dissipation and excitation terms are relatively large (Franaszek and Simiu, 1995).

We consider for definiteness the equation

$$\ddot{z} = -V'(z) + u(t) + \epsilon[\gamma G(t) - \beta\dot{z}] \quad (1)$$

$$2\pi\Psi(\omega) = \begin{cases} 0.03990 \ln(\omega) + 0.12829 & 0.04 \le \omega \le 0.4 \\ 0.05755 \ln(\omega) + 0.14493 & 0.4 \le \omega \le 1.2 \\ -0.38301[\ln(\omega)]^2 + 1.06192 \ln(\omega) - 0.02941 & 1.2 \le \omega \le 15.4 \end{cases} \quad (8)$$

where $V(z)$ is a potential function, $u(t) \equiv -\epsilon\gamma_c G_c(t)$ is a stochastic control input, and β, γ, γ_c are positive constants. We assume: (i) $V(z)$ has the shape of a multiple well, so that the unperturbed system has a center at the bottom of each well and a saddle point at the top of the barrier between two adjacent wells, and the stable and unstable manifolds emanating from the saddle point at the top of the potential barrier between two adjacent wells are homoclinic or heteroclinic; and (ii) ϵ is sufficiently (though not asymptotically) small. Finally we assume $G(t)$ is a random process with spectral density $2\pi\Psi(\omega)$. As a typical example belonging to the class of systems just described we consider the Duffing-Holmes equation, which has potential

$$V(z) = z^4/4 - z^2/2, \quad (2)$$

homoclinic orbits with coordinate

$$z_s(t) = \pm(2)^{1/2} \operatorname{sech}(t), \quad (3)$$

and a modulus of the Fourier transform of the function $h(t) = \dot{z}_s(-t)$

$$S(\omega) = (2)^{1/2}\pi\omega \operatorname{sech}(\pi\omega/2). \quad (4)$$

The function $S(\omega)$ is known as the Melnikov scale factor (Beigie et al., 1991). We also note that

$$c \equiv \int_{-\infty}^{\infty} \dot{z}_s^2(\tau)d\tau = \tfrac{4}{3}. \quad (5)$$

Associated with Eq. (1) is a Melnikov process with the expression

$$M(t) = -\beta c + \gamma \int_{-\infty}^{\infty} h(\tau)G(t-\tau)d\tau. \quad (6)$$

(Frey and Simiu, 1993). Any realization of the Melnikov process represents the distance between the stable and unstable manifolds of Eq. (1) ($\epsilon \ne 0$) corresponding to a realization of the random process $G(t)$.

The mean zero upcrossing time τ_u of the Melnikov process induced by the excitation is a measure of the mean time of exit from a well, τ_e (Sivathanu et al., 1996). For any given system, τ_u can be increased by adding to the excitation $\epsilon\gamma G(t)$ a control force $u(t) = -\epsilon\gamma_c G_c(t)$, where $0 < \gamma_c < \gamma$. A trivial choice of the open-loop control force would be $G_c(t) \equiv G(t)$. For this choice, the ratio between the average power of the exciting force and the average power of the control force is $Q = \gamma^2/\gamma_c^2$. We seek to use the information contained in the Melnikov relative scale factor $S(\omega)$ to obtain open-loop control forces that would achieve results comparable to those achieved by the trivial control, but with considerably more effectiveness.

From Eq. (6) it follows that the spectral density of the Melnikov process for the uncontrolled system is

$$2\pi\Psi_M(\omega) = S^2(\omega)[2\pi\Psi(\omega)] \quad (7)$$

where $S(\omega)$ is the modulus of the Fourier transform of $h(t)$, and $2\pi\Psi(\omega)$ is the spectral density of the random process $G(t)$. To illustrate Eq. (7), we consider the Duffing-Holmes equation, for which $S(\omega)$ is given by Eq. (4). Let us consider a force $G(t)$ with spectral density

(Fig. 1). To a first approximation this spectrum is representative of low-frequency fluctuations of the horizontal wind speed (Van der Hoven, 1957). The functions $S^2(\omega)$ and $2\pi\Psi(\omega)S^2(\omega)$ are represented in Figs. 2(a) and 2(b), respectively. Figures 1 and 2 show that, owing to the shape of $S(\omega)$—which plays the role of an admittance function—only part of the frequency components of the excitation $G(t)$ contribute significantly to the spectral density of the uncontrolled system's Melnikov process (for example, components with frequencies $\omega > 4$ are suppressed; components with frequencies $2.5 < \omega < 4$ are very strongly reduced).

The following approach appears reasonable. Instead of $G_c(t) \equiv G(t)$, it would be more efficient to apply a control force obtained by filtering out from the function $G(t)$ those frequency components that do not contribute significantly to the spectral density $\Psi_M(\omega)$. The advantage of this approach over the trivial approach $G_c(t) \equiv G(t)$ is that, for the same control power, it would result in smaller ordinates of the controlled system's Melnikov process and in a lower mean exit rate—since those smaller ordinates entail smaller chaotic transport across the pseudoseparatrix (Beigie et al., 1991; Frey and Simiu, 1993).

Like its trivial counterpart, the approach just described is not feasible owing to practical limitations on the operation of the control system. These limitations entail non-zero time lags between sensing of a signal and the actuator response. In addition, practical filters may entail other inefficiencies. Nevertheless, our approach can be effective, as is illustrated by the numerical simulations presented in the next section.

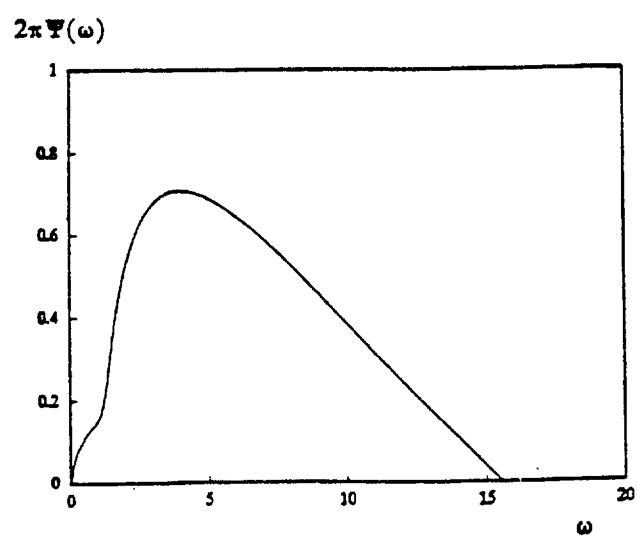

Fig. 1 Spectral density of excitation

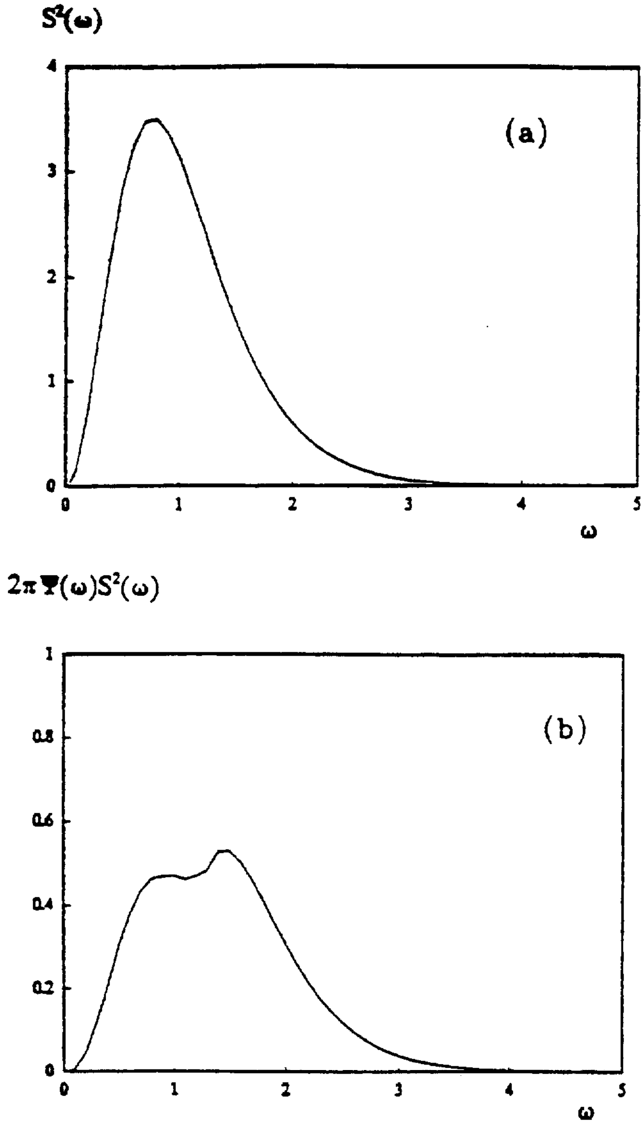

Fig. 2 *(a)* **Square of Melnikov relative scale factor;** *(b)* **spectral density of Melnikov process**

Numerical Simulations

We considered the Duffing-Holmes equation (Eqs. (1) and (2)), $\epsilon = 0.1$, $\beta = 0.45$. We examined two cases: (1) $2\pi\Psi(\omega)$ is given by Eq. (8); (2) $2\pi\Psi(\omega) = 2\pi/5$ for $0 < \omega < 5$ and $2\pi\Psi(\omega) = 0$ otherwise. We first estimated by numerical simulation the mean exit rate for the uncontrolled system. We then estimated the mean exit rate for the system with control forces. We considered four types of control force:

(*a*) Control type (*a*) seeks to counteract the excitation by applying, with lag t_0, a control force proportional and of opposite sign to $\epsilon\gamma G(t)$. The smaller the lag t_0, the more effective the control.

(*b*) Control type (*b*) utilizes the information provided by the Melnikov scale factor $S(\omega)$ as follows. Consider for example excitation case 1. Figure 2 shows that spectral components with frequencies $0 \le \omega \le \omega_1$, where $\omega_1 < 0.3$, say, and frequencies $\omega > \omega_2$ where $\omega_2 = 2.5$, say, contribute little to the spectral density of the Melnikov process. In other words, the spectral components inside (outside) the interval (ω_1, ω_2) are effective (ineffective). Control type (*b*) differs from control (*a*) in that all ineffective components are suppressed.

(*c*) Control type is similar to control type (*a*), except that the signal $-\epsilon\gamma_c G_c(t)$ is first passed through a realistic, practical filter with impulse response function shown in Fig. 3 ($A = 0.1$, $B = 2.25$)

(*d*) Control type (*d*) is similar to control type (*c*), except that all ineffective Fourier components are suppressed from the output of the filter, while leaving the other components unchanged.

In all the simulations we assumed a time lag $t_0 = 0.1$. The frequencies ω_1 and ω_2 defining the intervals over which inefficient components were filtered out in (*b*) and (*d*) were chosen by examining the spectral densities of the Melnikov processes. The choices were $\omega_1 = 0.3$, $\omega_2 = 2.5$ (excitation case 1) and $\omega_1 = 0.3$, $\omega_2 = 0.2$ (excitation case 2).

The strength of the control force was assumed to be $\gamma_c = 0.5\gamma$ for control types (*b*) and (*d*). For control types (*a*) and (*c*) γ_c was chosen so that the control forces have the same average power (i.e., the same variance) as the control forces for type (*b*) and (*d*), respectively. This choice yielded, in excitation case 1, $\gamma_c = 0.167\gamma$ for type (*a*) and $\gamma_c = 0.197\gamma$ for type (*c*) and, in excitation case 2, $\gamma_c = 0.292\gamma$ for type (*a*) and $\gamma_c = 0.345\gamma$ for type (*c*).

The filter of Fig. 3 has transfer function $\Lambda(\omega) = R(\omega) + jI(\omega)$, where

$$R(\omega) = r^2(A\omega/2)\cos(A\omega) - r^2(B\omega/2)\cos(2A + B)\omega \quad (9a)$$

$$I(\omega) = -r^2(A\omega/2)\sin(A\omega) + r^2(B\omega/2)\sin(2A + B)\omega. \quad (9b)$$

$r(x) = \sin(x)/x$ (Frey and Simiu, 1996). Equations (9) were obtained from expressions given in Papoulis (1962). We show in Fig. 4 the dependence on frequency of the filter gain and phase.

The numerical simulations were performed by the adaptive step-size Runge-Kutta method. The realizations of the excitation process $G(t)$ were simulated by sums of twenty-five sine and cosine terms with equally spaced frequencies and amplitudes distributed normally with zero mean and variance $2\pi\Psi(\omega)\Delta\omega$, where $\Delta\omega$ is the frequency increment (Rice, 1954). For each realization the initial points were chosen randomly and the trajectories were integrated for a time interval $T_{tot} = 1000T$, where $T = 2\pi/\omega_{max}$ and ω_{max} is the maximum energy-containing frequency of the spectrum of $G(t)$. The number of zero crossings was counted for each of a total of 800 realizations. For this number of realizations the error in the estimation of the main upcrossing rate was found to be about 0.5 percent. A similar procedure was applied to the controlled system. The results are shown for cases 1 and 2 in Figs. 5 and 6, respectively, where $\sigma \equiv \epsilon\gamma$.

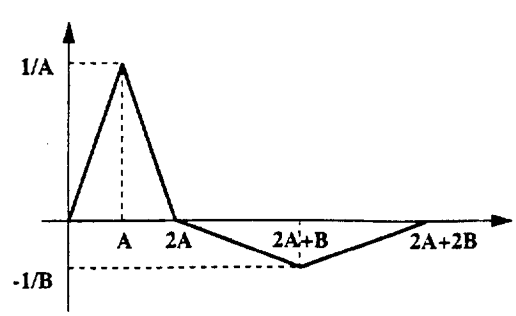

Fig. 3 Impulse response of a two-parameter filter with initial response and recoil

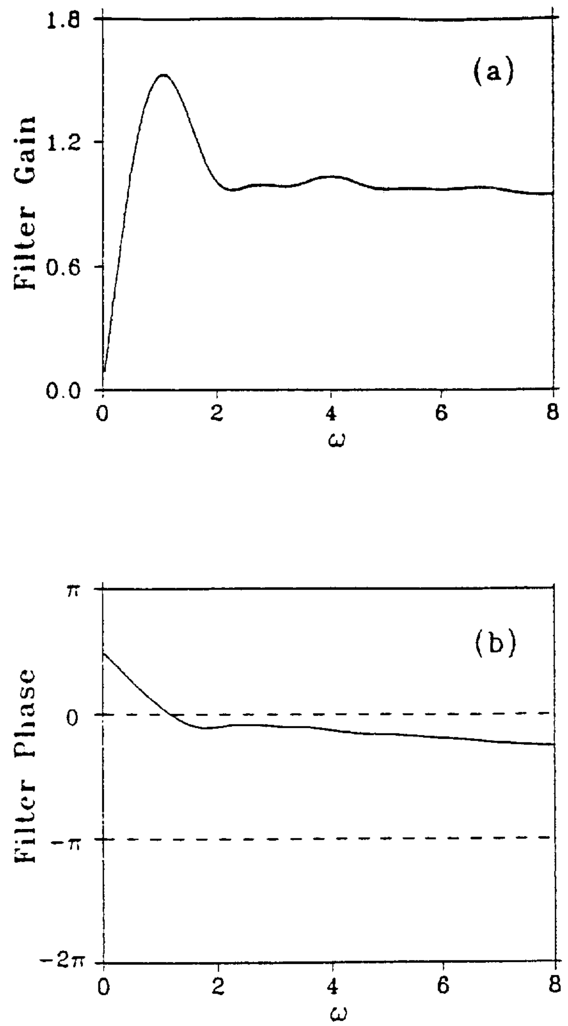

Fig. 4 (*a*) Gain and (*b*) phase angle for filter of Fig. 3 with A = 0.1 and B = 2.75

Discussion

We now compare mean escape rates induced by control forces modified to take advantage of the system's Melnikov properties with rates induced by those forces' unmodified counterparts. Recall that the control forces corresponding to curves (b) and (d) in Figs. 5 and 6 have the same average power as those corresponding to curves (a) and (c), respectively, and that they were obtained from the latter by using Melnikov theory to eliminate inefficient components. It is seen that this procedure is useful for both cases 1 and 2. The benefits tend to be stronger for case 1, for which $S^2(\omega)$ renders ineffective a larger proportion of the total power of the excitation process $G(t)$ than is the case for case 2 (see Figs. 5 and 6, and recall that for case $2\Psi_0(\omega) = 0$ for $\omega > 5$). For example, Fig. 5 shows that, given the external excitation $\epsilon\gamma$ = 0.15, for case 1 the escape rate reduction due to the use of a control force type (b) is about 20 times larger than that due to a control force type (a) having the same average power; while control force type (d) is about 5 times more effective than control force type (c) with the same power. Note that the effectiveness of the control force increases as $\epsilon\gamma$ decreases.

To conclude, the exploratory numerical simulations presented in this section suggest that controls based on the information contained in the Melnikov transfer function can help to stabilize efficiently a system subjected to random excitation. The degree to which an efficient Melnikov-based open-loop control can be accomplished in practice depends upon the system under consideration (i.e., its Melnikov characteristics), the spectral density of the excitation, and the characteristics of the filters used to obtain the control force.

Conclusions

A Melnikov-based open-loop approach to the control of a class of nonlinear stochastic systems was proposed. The aim of the proposed approach is to achieve a relatively efficient stabilization of the system. Exploratory numerical simulations suggested that the information contained in the Melnikov relative scale factors can help to achieve this objective. It is emphasized that our calculations fully accounted for the nonlinearity of the systems at hand.

The degree to which an efficient Melnikov-based open-loop control can be accomplished in practice depends upon the system under consideration (i.e., upon its Melnikov scale factors), the spectral density of the excitation, and the quality of the filter design. The intent of this paper is not to study the filter problem in the context of Melnikov-based open-loop control. Rather, it is to draw the attention of control specialists to the approach

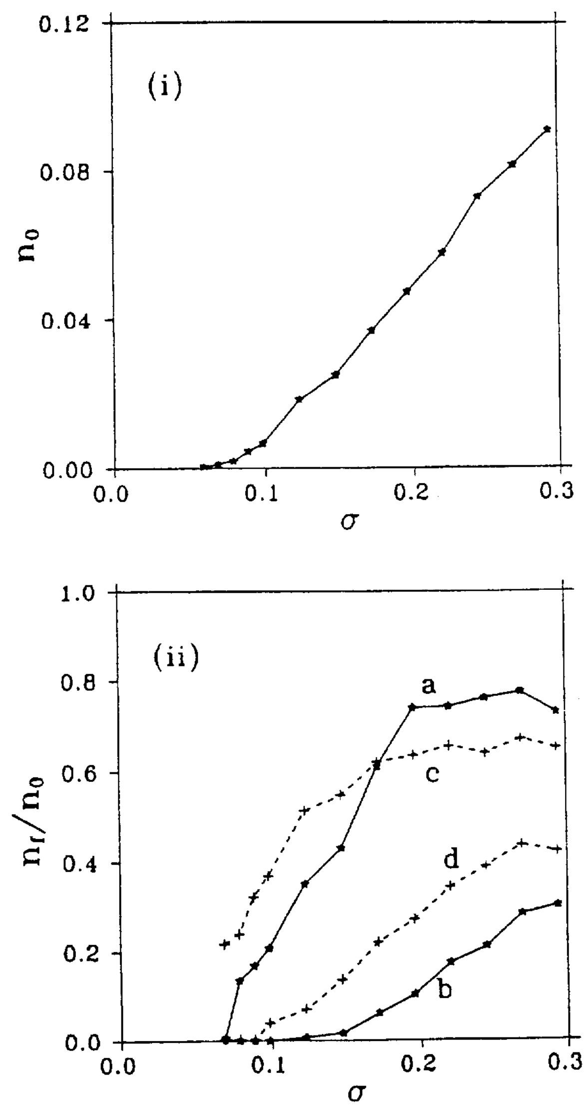

Fig. 5 Case 1: (i) Escape rate n_0 for uncontrolled oscillator subjected to noise $\sigma = \epsilon\gamma$; (ii) ratio n_t/n_0 between escape rate of controlled and uncontrolled system. Curves (*a*), (*b*), (*c*), (*d*) are described in the text; $\sigma = \epsilon\gamma$.

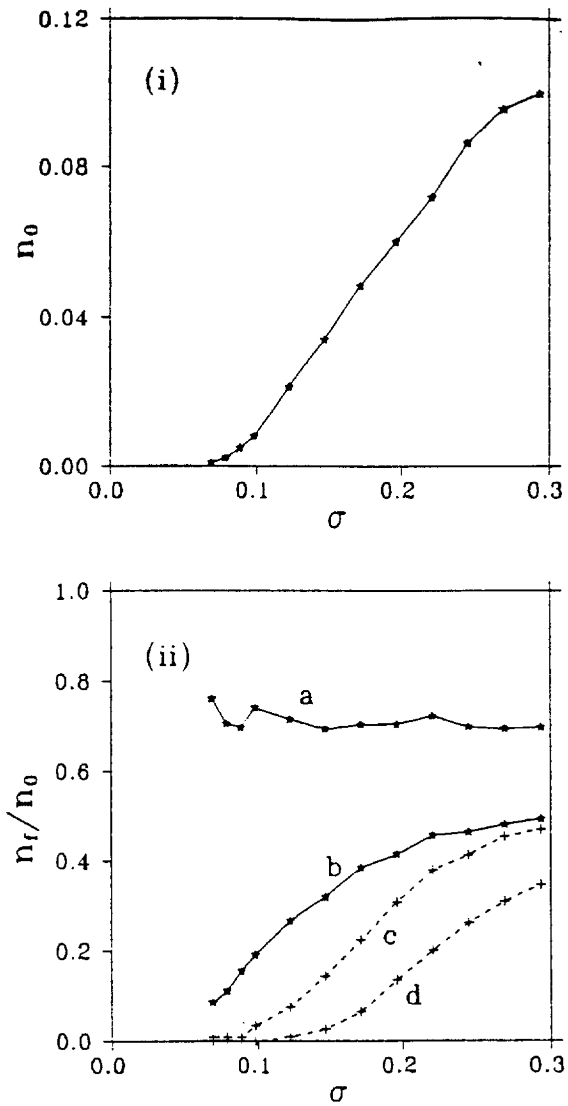

Fig. 6 Case 2: same legend as for Fig. 5

proposed herein, in the belief that, whether used singly or as a component of a more complex control strategy, it may become a useful addition to the current body of nonlinear control theory and practice.

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