Engineering Foundation First International Conference on High Strength Concrete, Keauhou-Kona, Hawaii, July 13-18, 1997.

SHEAR STRENGTH OF PC BEAMS: PARAMETRIC STUDY OF SHEAR FRICTION AND CONCRETE SOFTENING

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ABSTRACT

The Modified Compression Field Theory (MCFT) is used to study the effect of shear friction and biaxial softening on the computed shear strength of prestressed concrete (PC) beams. According to the MCFT, a decrease in shear friction within the range of experimental data, as found in high-strength concrete (HSC), can lower the shear strength of beams with minimum shear reinforcement by 15 % to 30 % depending on the method of estimation. For high levels of shear reinforcement, the effect is minor and may not be of practical significance.

In addition, a comparison is presented of different relationships used to represent the biaxial compression-tension strength of reinforced concrete. For PC beams, some theories of biaxial softening of concrete do not predict concrete crushing even for very high deformations, but rather show significant shear force gain after stirrup yielding and crack slipping. However, the first peaks of the shear force versus crack width curves, which occur close to stirrup yielding and crack slipping, are within 102 % - 111 % of the experimental shear strength.

INTRODUCTION

The importance of aggregate interlock, or shear-friction, across shear cracks has been recognized for quite some time as one of the mechanisms of shear resistance in reinforced concrete (RC) beams. ACI code provisions for shear (ACI 318-95 1) do not take explicit account of shear friction, but rather lump it together with other factors, such as dowel effect and the shear carrying capacity of the compressed part of the beam, into the concrete contribution term V_c.

In the last 20 years, more rational methods for shear strength calculation have been able to explicitly account for the contribution of shear friction across cracks in resisting shear. One noteworthy method, which has been adopted in the Canadian Code (CSA A23.3-94 2), the Norwegian Code (NS 3473 E 1992 3) and the AASHTO LRFD Bridge Design Specifications (AASHTO 1994 4), is the Modified Compression Field Theory or MCFT (Vecchio and Collins 5, Collins and Mitchell 6).

Another aspect of diagonal shear cracks is that they also weaken the concrete struts. The presence of transverse tensile stress and strain lowers the concrete compressive strength below its uniaxial strength (softening). The MCFT provides a means to evaluate the effect of this softening on the shear strength of PC beams.

Following reviews of the MCFT, and other works on shear friction and biaxial softening, this paper[a] presents the results of a parametric study that determines the effects of changes in shear friction and concrete softening on the shear strength of PC beams, as predicted by the MCFT.

REVIEW OF THE MODIFIED COMPRESSION FIELD THEORY (MCFT)

The MCFT is a rational theory capable of predicting the strength of reinforced and prestressed concrete beams under shear and axial loading. It is rational in the sense that it satisfies equilibrium of forces and moments, compatibility of displacements, and the stress-strain relationships of concrete and reinforcing steel. One of the simplifying assumptions of the MCFT is that the principal directions of stress and strain coincide, which is only true within ± 10° for cracked reinforced concrete. According to the MCFT, the shear strength V of a PC beam is the sum of a stirrup contribution V_s, which is based on the variable angle truss model, a concrete contribution V_c, and a component of the prestressing force V_p.

The concrete contribution V_c is the shear resisted by tensile stresses f_{c1} acting between diagonal cracks and is a function of the shear that can be transmitted across cracks by aggregate interlock. Indeed, after yielding of the transverse reinforcement, transmittal of *tension* across cracks requires local shear stresses τ along cracks. The ability of the crack interface to transmit shear stress τ depends strongly on the crack width w. Vecchio and Collins (5) allowed for the possibility of local *compressive* normal stress σ across cracks. Based on Walraven and Reinhardt's (7) experimental results, they suggested the following parabolic equation to relate τ to σ:

$$\frac{\tau}{\tau_{max}} = 0.18 + 1.64\frac{\sigma}{\tau_{max}} - 0.82\left(\frac{\sigma}{\tau_{max}}\right)^2 \qquad (1)$$

$$\text{with} \quad \tau_{max} = \frac{\sqrt{f'_c}}{0.3 + \dfrac{24\,w}{a + 16}} \quad \text{N, mm} \qquad (2)$$

It turns out that the values of σ in beams failing in shear is negligible and Eq. (1) was simplified in later versions of the MCFT to (Collins and Mitchell 6):

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[a] A more complete version of this paper, which includes discussion of RC beams as well as PC beams and Code implications, is about to be submitted to the ACI Structural Journal for publication and is available upon request.

$$\tau = 0.18\,\tau_{max} \tag{3}$$

Thus the shear stress that can be transmitted along cracks is related to concrete strength, crack width and aggregate size.

The MCFT assumes a parabolic relationship (Hognestad 8) to describe the stress-strain behavior of concrete in compression:

$$\frac{f_{c2}}{f_{c2max}} = 2\left(\frac{\varepsilon_2}{\varepsilon_0}\right) - \left(\frac{\varepsilon_2}{\varepsilon_0}\right)^2 \tag{4}$$

A softening parameter β is defined as the ratio of f_{c2max} to the uniaxial cylinder compressive strength f'_c.

$$\beta = \frac{f_{c2max}}{f'_c} = \frac{1}{0.80 + 0.34\,\varepsilon_1/\varepsilon_0} \leq 1.0 \tag{5}$$

Equation (5) was derived from tests of biaxially loaded panels, with a mean ratio of test values to equation predictions of 0.98 and a coefficient of variation for the same ratio of 0.16. For a strain ε_0 equal to 0.002, Eq. (5) becomes:

$$\beta = \frac{1}{0.80 + 170\,\varepsilon_1} \tag{6}$$

Equation (6) is used in the Canadian Code (CSA 1994 2). Thus, the principal compressive stress in the concrete web f_{c2} is a function, not only of the principal compressive strain ε_2, but also of the co-existing principal tensile strain ε_1.

REVIEW OF SHEAR FRICTION

Walraven (9), Walraven and Reinhardt (7), and Walraven, Frénay and Pruijssers (10) performed some important work at Delft University of Technology on the constitutive relations of cracks in concrete loaded in shear. Their work accounts for aggregate interlock, dowel action and axial tension of the reinforcement crossing a crack, combines experiment and theory, and shows good agreement between the two. More recently, Walraven (11), and Walraven and Stroband (12) showed that high-strength concrete (HSC) exhibits a marked reduction in shear friction compared with normal-strength concrete (NSC). These studies showed that the shear stress that can be transferred across a crack depends on the compressive stress across the crack, the crack width, the crack slip, the strength of the mortar and the concrete, the coefficient of friction between mortar and aggregate, the maximum size and the volumetric percentage of aggregate.

Besides the MCFT, other theories of beam shear strength also use the Delft experimental results to account for shear friction. The works of Reineck (13, 14), Kupfer and Bulicek (15), Kupfer, Mang and Karavesyroglou (16), Prisco and Gambarova (17), Dei Poli, Prisco and Gambarova (18), Bažant and Gambarova (19) are noteworthy.

REVIEW OF CONCRETE SOFTENING

The web of a reinforced concrete beam under shear is in a state of biaxial tension-compression. The presence of simultaneous transverse tensile strain leads to a reduction of the compressive strength of cracked concrete. This "softening" behavior has been investigated in panel tests.

Vecchio and Collins (20) reviewed various models of compression softening of cracked reinforced concrete panels due to transverse tension and updated their model, Eqs. (5) and (6), in two different ways. They changed their base uniaxial stress-strain curve to Thorenfeldt's (21) curve, which is more appropriate for HSC (more linear in its pre-ultimate response) than Hognestad's parabola. Model *A* uses "strength" and "strain softening", i.e., both peak stress and its corresponding strain decrease under biaxial stress, whereas Model *B* uses only strength softening.

Kollegger and Mehlhorn (22, 23) concluded that the effective compressive strength did not reduce beyond 0.8 f_c' and that the prime influencing factor appeared to be the principal tensile stress f_{c1} rather than the principal tensile strain ε_1. A value of 0.8 f_c' is also used by Reineck (13, 14) as the lower limit for the strength of web struts. Similarly, Kupfer, Mang and Karavesyroglou (16) used an experimental softening factor of 0.85 coupled with a sustained load factor of 0.80:

$$f_{c2} = 0.80 \times 0.85 \times f'_c \approx 2/3\ f'_c \qquad (7)$$

Kupfer and Bulicek (15) used a factor of 0.85 to account for sustained load, a factor of 0.75 to account for irregular crack trajectory, and a factor of (1 - f'_c/ 250) to account for uniaxial prism strength:

$$f_{c2} = f'_c \times 0.85 \times 0.75 \left(1 - \frac{f'_c}{250}\right) \quad \text{MPa} \qquad (8)$$

Miyahara, Kawakami and Maekawa (24) proposed a softening model based on the principal tensile strain. The degree of softening is much less than that predicted by Vecchio and Collins. Tanabe and Wu (28) reviewed other Japanese experimental results for biaxial tension-compression: Maekawa and Okamura studied concrete softening based on measurements of reinforced cylindrical specimens under axial compression and internal pressure, whereas Shirai performed tests of small reinforced panels. Further Japanese work on biaxial softening includes Shirai and Noguchi (25), Mikame, Uchida and Noguchi (26) and Ueda et al. (27).

Belarbi and Hsu (29) used Hognestad's parabola as a basis and suggested one softening parameter for stress and another for strain. In a later paper, Belarbi and Hsu (30) presented the results of tests of 22 panels 1400 x 1400 x 178 mm under biaxial tension-compression. Prisco and Gambarova (17) accounted for the effects of transverse reinforcement in tension by reducing the concrete strength to 0.75 f_c' or as a function of ε_1 as formulated by Hsu (31).

As can be seen from the wide variety of formulations, a consensus has yet to be reached among researchers, on whether the concrete softening parameter is constant, or depends on the average principal tensile stress or strain. This paper reports on a study to determine how these various formulations affect beam shear strength. The tool used for this purpose is the MCFT.

PARAMETRIC STUDY

As mentioned in the preceding review, the MCFT accounts for shear transfer across cracks and concrete softening due to the biaxial state of tension-compression in the web of beams loaded in shear. Since different formulations for shear friction and concrete softening exist, a parametric study is performed using the MCFT to determine the influence of these two factors on beam shear strength. For this purpose, computer program SHEAR and an example beam (Fig. 1) are adapted from Collins and Mitchell (6). SHEAR can predict the shear force-crack width response of reinforced or prestressed concrete beams subjected to shear or shear combined with axial load. At each load step, the user inputs a value of principal tensile strain ε_1. The program assumes a strut angle θ, then computes strains, loads, crack widths, etc. according to a 17-step procedure developed by Collins and Mitchell (6) to implement the MCFT. In particular, Eqs. (3), (4) and (6) are coded into the version of SHEAR supplied in Collins and Mitchell's book (6). If convergence is not achieved, another value of θ is assumed. The program stops when equilibrium cannot be achieved after a prescribed number of iterations, due to concrete crushing or all reinforcements yielding.

Shear Friction

As mentioned above, shear friction enters into the MCFT as a parameter $F = \tau / \tau_{max} = 0.18$ with τ_{max} a function of crack width w and maximum aggregate size a, Eqs. (2) and (3). For the parametric study, this shear friction parameter was varied from a low value of $0.35 \times 0.18 = 0.063$ to a high value of $1.5 \times 0.18 = 0.27$. The program SHEAR was modified by varying F.

Figs. 2a, b and c show the computed shear force V versus crack width w relationship for a concrete strength of f'_c = 38.6 MPa and for various combinations of shear friction parameter (F = 0.063, 0.18, or 0.27) and shear reinforcement ratio (ρ_v = 0.12, 0.28, 0.61, or 1.11 %).

The shear reinforcement comprises #2 bars (smooth, ϕ = 6 mm) at 356 mm or 152 mm spacing, #3 or #4 bars (deformed, ϕ = 9.5 or 13 mm) at 152 mm spacing. As the reinforcement varies, so does its crack control characteristics (s_{mx}, s_{mv}) which must be entered into the program (Table 1). The shear reinforcement obeys ACI design guidelines for minimum quantity and maximum spacing.

Table 1 Stirrup and Crack Spacing for PC Beam

Stirrup bar #	s mm	ρ_v %	s_{mx} mm	s_{mv} mm
2 (smooth)	356	0.12	414	1252
2 (smooth)	152	0.28	414	602
3	152	0.61	414	295
4	152	1.11	414	246

The base case (F = 0.18, ρ_v = 0.61 %) corresponds to a PC beam tested by Arbesman and Conte (Fig. 1) and used as an example by Collins and Mitchell (6). The measured strength of the beam was V_x = 430 kN versus a prediction of 473 kN by Collins and Mitchell (6). In all cases, failure was by diagonal compression (concrete crushes, symbol *c* in curves), preceded by stirrup yielding (symbol *y*) and crack slipping (symbol *s*). The program was run until beam failure, even after crack widths had reached unrealistic values (the range of shear friction laws only extends to $w \leq$ 1.5 mm), to show the increase in ductility as the amount of shear reinforcement (always above the minimum required by ACI 318) or the shear friction parameter decreases. Also, as crack widths increase, the *V-w* curves for various friction parameters approach one another, as they should, since shear friction approaches zero, Eq. (2). Two types of behavior can be observed (Fig. 2):

- For high and medium shear reinforcement ratio (ρ_v > 0.3 %), the *V-w* curve typically follows a linear path up to stirrup yielding or crack slipping (Fig. 2b). The latter occurs when tension in the concrete reaches a limit imposed by the shear reinforcement and shear friction across cracks :

$$f_{1limit} = \tau \tan \theta + \frac{A_v}{b_w s}(f_{vy} - f_v) \tag{9}$$

  For high friction (F=0.27), stirrups yield before cracks slip; for low friction (F=0.063), the order is reversed; and for medium friction (F=0.18), stirrup yielding and crack slipping occur simultaneously. Peaks of shear force V occur at initiation of crack slipping, although for low friction, it's only a local peak (V in this case reaches its global peak at large crack widths w > 2 mm).

- For low shear reinforcement, the shear force reaches a peak at a small crack width (w < 0.05 mm), then drops precipitously when concrete starts to crack. For ρ_v = 0.28 %, after the initial drop, the load recovers and increases to a maximum until cracks start to slip, at which point, it starts to decrease (Fig. 2c). For ρ_v = 0.28 % and low friction, and for ρ_v = 0.12 %, there is no load increase after the initial sharp drop at initial cracking. However, for ρ_v = 0.12 % and high or medium friction, a change of slope is still noticeable at initial crack slip.

Because the shapes of the *V-w* curves vary, several methods are used to compare the values of V obtained for various parameters:

- Peaks of V are used. For ρ_v = 0.12 %, "peaks" degenerate to points of sudden change in negative slope.
- Values of V at w = 1 mm are selected.
- Where a linear part exists (prior to crack slipping or stirrup yielding), the values of V at the end of the linear range are selected.
- Intersections of the curves with a straight line parallel but offset with respect to the linear part by w = 1mm are also used.
- Finally, where a linear part does not exist (e.g., for ρ_v = 0.12 %), intersections of the curves with a straight line passing through the value of V at w = 1 mm for medium friction and parallel to the initial slope of the closest set of curves with a definable initial slope (here ρ_v = 0.28 %) are used.

Results (Table 2 and Fig. 2) show that:

- As the shear reinforcement ratio decreases, the effect of shear friction increases. This is to be expected since, as shear reinforcement decreases, the proportion of shear load carried by shear friction increases. For a shear friction parameter of 35% of the base case, as in HSC compared to NSC, the shear force V at or near its peak is 15 % to 30 % lower than for the base case, depending on the method of estimation.

- Two cases were run for the PC beam (for ρ_v = 1.11 % , F = 0.18 or 0.063) using the 1986 version of the MCFT, which has a more elaborate shear friction law, Eq. (1), compared with the 1991 version, Eq. (3). The results of the two versions are indistinguishable from one another, i.e., the normal compressive stress σ across shear cracks is negligible.

- Failure by concrete crushing is predicted to occur at high w (very wide cracks), much higher than the range of Walraven's experimental data ($v \leq$ 2 mm, $w \leq$ 1.5 mm).

Table 2 Variation of Shear Strength V of a PC Beam for Various Values of Shear Friction F and Stirrup Ratio ρ_v.

F	0.27			0.18			0.063			Method for shear strength
ρ_v %	θ°	V (kN)	%	θ°	V (kN)	%	θ°	V (kN)	%	
1.11	29.3	653.1	103	29.5*	632.7*	100	28.6*	602.2*	95	1 mm offset
	30.9	646.7	100	30.9*	645.0*	100	29.9*	593.4*	92	linear limit
	29.5	655.7	102	30.9*	645.0*	100	28.3*	604.5*	94	peak
0.61	24.3	477.3	105	23.9	456.5	100	23.2	428.3	94	1 mm offset
	26.8	492.8	104	26.5	472.8	100	25.6	414.1	88	linear limit
	25.8	500.5	106	26.5	472.8	100	22.7	429.7	91	peak
0.28	20.8	318.7	107	20.5	297.3	100	19.9	261.8	88	1 mm offset
	21.6	329.6	111	20.5	297.3	100	19.0	258.0	87	w = 1 mm
	22.6	348.9	109	22.2	321.0	100	22.4	281.1	86	peak
0.12	17.2	222.2	110	16.4	201.5	100	14.9	169.3	84	1 mm offset
	18.0	234.3	116	16.4	201.5	100	14.2	163.6	81	w = 1 mm
	21.5	284.3	97	22.6	292.1	100	17.1	194.7	67	slope

* Results obtained using either Eq. (1) or (3).

Biaxial Softening

The computer program SHEAR was modified by replacing Eqs. (4) and (6) with various stress-strain relations and biaxial softening models. The program was used to predict the strength of the experimental beam (Fig. 1) and was stopped after large crack widths were attained (about 20 mm) or when equilibrium could no longer be sustained because of concrete crushing (diagonal compression) or all reinforcements yielding.

The curve of shear force versus crack width ceases to be linear shortly after stirrups yield and cracks slip. Peak forces (local peaks in some cases) occur near that point and are compared in Table 3. Two types of *V-w* behavior are observed for the various softening models (Table 3 and Fig. 3):

- Significant post-linear strength gain is predicted by the models of Kollegger (22, 23), Okamura (28), Miyahara (24) and Shirai (28), which predict no concrete crushing (failure is by excessive deformation); and by the models of Ueda (27) and Noguchi (25), which predict fairly similar behavior, namely concrete crushing after considerable post-linear strength and wide cracks.

- No post-linear strength gain is predicted by the models of Collins (6), Vecchio-B (20), and Hsu (30).

- Despite these differences, the first peaks of shear force, which occur close to stirrup yielding and crack slipping, are within 102-111 % of the experimental shear strength of the PC beam for all models.

Table 3 Peak Shear Force for Various Biaxial Softening Laws
(ρ_v = 0.61 %, F = 0.18. Subscript L for end of linear range.)

Model	V_L (kN)	V_L / V_X (%)	w_L (mm)
Kollegger (22,23)	476	111	0.8
Shirai (28)	455	106	0.6
Okamura (28)	465	108	0.6
Miyahara (24)	438	102	0.6
Noguchi (25)	460	107	0.6
Ueda (27)	477	111	0.8
Hsu (30)	471	110	0.6
Vecchio-B (20)	459	107	0.6
Collins (6)	473	110	0.7

CONCLUSION

Relationships for shear friction and biaxial softening of concrete used in various beam shear theories vary widely. The Modified Compression Field Theory (MCFT) was used to study the effects of various shear friction and concrete softening formulations on the calculated shear strength of prestressed concrete beams. According to the MCFT, a decrease in shear friction within the range of experimental data, as found in high-strength concrete, lowers the shear strength of beams with low shear reinforcement by 15 % to 30 %, depending on the method of estimation. For high levels of shear reinforcement, the effect is minor and may not be of practical significance.

In addition, a comparison is presented of different relationships used to represent the biaxial compression-tension strength of reinforced concrete. For PC beams where the prestressing cables do not fail, some theories of biaxial softening of concrete do not predict concrete crushing even for very high deformations. However, the first peaks of shear force, which occur close to stirrup yielding and crack slipping, are within 102 % - 111 % of the experimental shear strength of the prestressed concrete beam.

ACKNOWLEDGMENT

Mike Collins, Julio Ramirez and Nick Carino reviewed earlier drafts of this paper and offered valuable suggestions; Wilasa Vichit-Vadakan helped with some of the computer runs and plots; and Long Phan served as Division Reviewer. To all, the author extends his thanks.

NOTATION

A_v = area of shear reinforcement
a = maximum size of aggregate
b_w = beam width
F = τ / τ_{max} = friction parameter
f_c' = concrete cylinder strength or uniaxial compressive strength
f_{c1} = principal tensile stress in concrete web
f_{c2} = principal compressive stress in concrete web
f_{c2max} = compressive strength of concrete panel in biaxial tension-compression
f_v = stress in shear reinforcement
f_{vy} = yield strength of shear reinforcement
s = stirrup spacing
s_{mx} = crack control characteristics of longitudinal reinforcement
s_{mv} = crack control characteristics of transverse reinforcement
V = shear force, shear strength
V_c = concrete contribution to shear strength
V_s = stirrup contribution to shear strength
V_p = vertical component of inclined prestressing tension
V_X = experimental shear strength
v = crack slip
w = crack opening
β = softening parameter
ε_0 = strain at maximum compressive stress for uniaxial compression
ε_1 = principal tensile strain in concrete

ε_2 = principal compressive strain in concrete
ε_{1L} = concrete tensile strain at which reinforcement at crack begins to yield
θ = strut angle
ρ_v = shear reinforcement geometrical ratio
σ = normal stress across a crack
τ = shear stress across cracks
τ_{max} = maximum shear stress transmitted across a crack
ϕ = bar diameter

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KEYWORDS

aggregate interlock; biaxial softening; building technology; prestressed concrete; reinforced concrete; shear; shear friction.

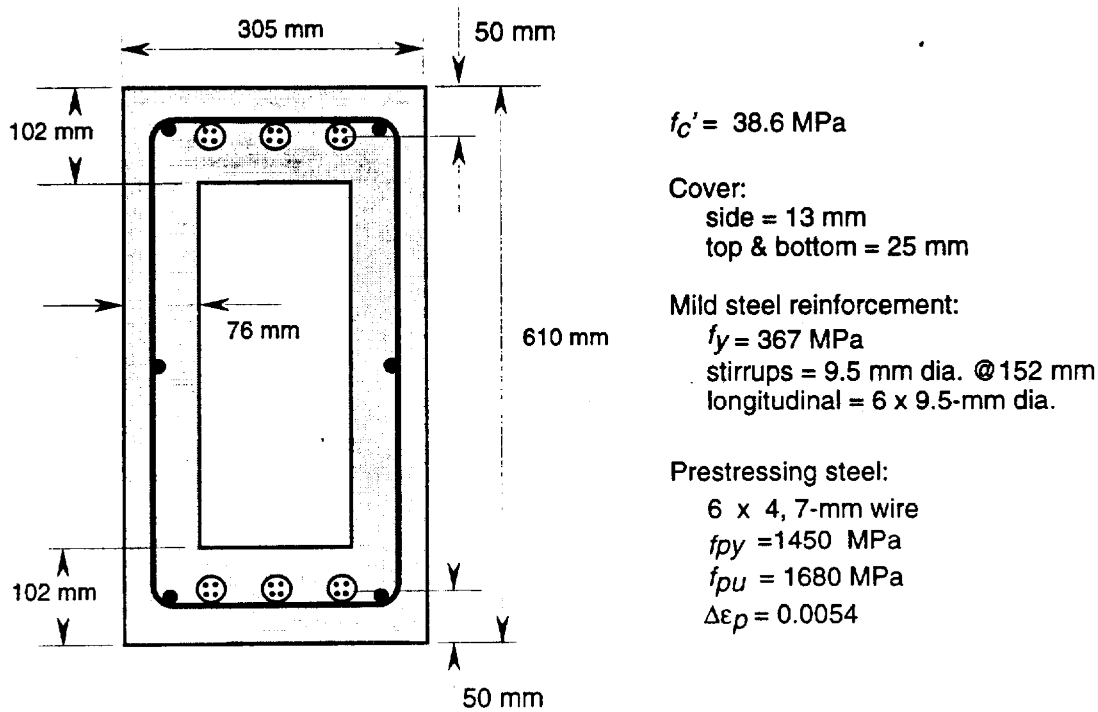


Fig. 1 - Cross section of PC beam CF1 used in parametric study (adapted from Collins and Mitchell 6)

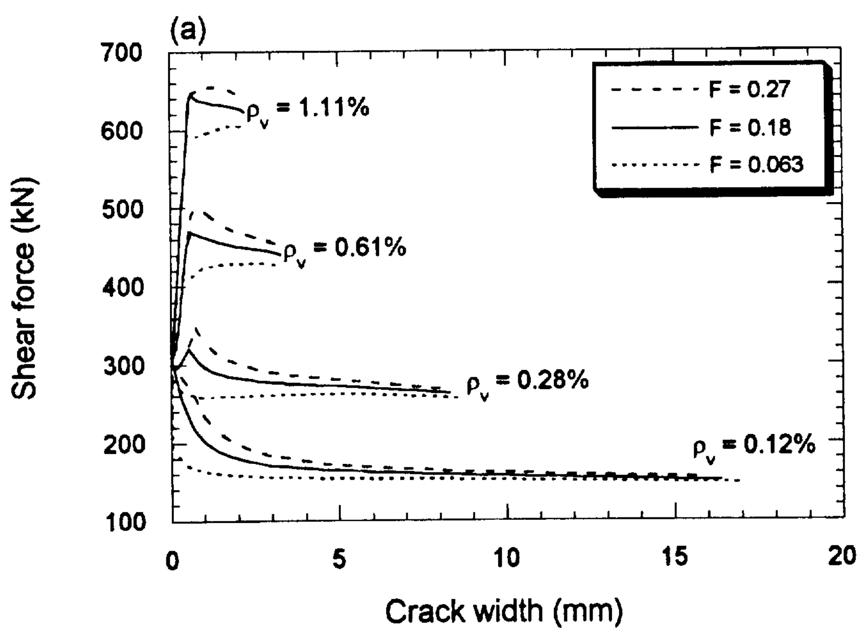
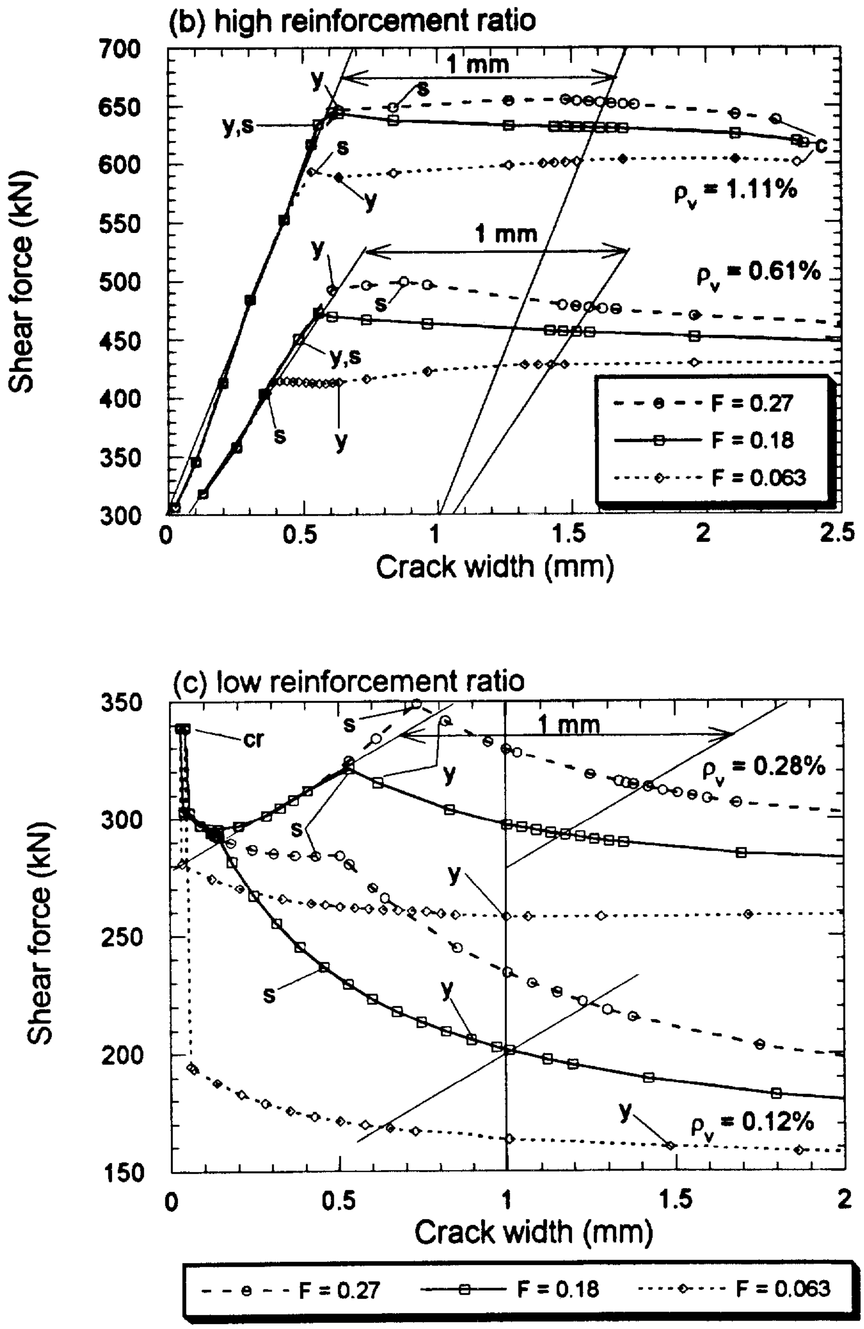


Fig. 2 - Effect of shear friction and shear reinforcement on PC beam behavior



cr = initial cracking; y = stirrups yield; s = cracks slip; c = concrete crushes

Fig. 2 - Effect of shear friction and shear reinforcement on PC beam behavior

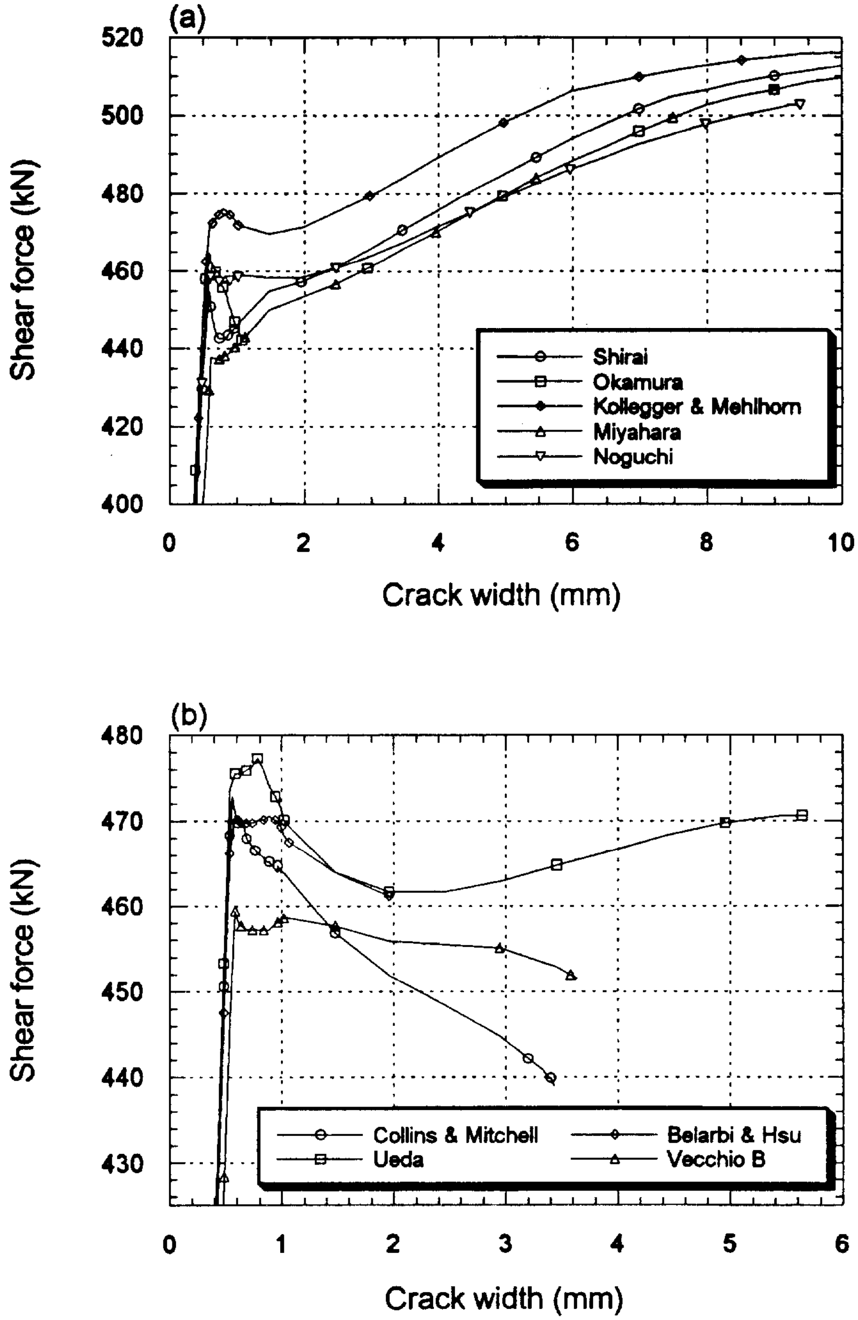


Fig. 3 - Effect of concrete biaxial softening on shear behavior of PC beam behavior