

OPTIMAL CONTROL OF AN EXCAVATOR BUCKET POSITIONING

by

E. Budny, M. Chłosta, W. Gutkowski ¹⁾

ABSTRACT.

Recently, there is an increasing interest in controlled excavation processes. However, the main attention, in research works, is paid to the bucket motion. This part of the process can be considered as a quasi static, kinematically induces process [3]. It means that dynamic effects, by dropping accelerations terms can be neglected. This is not a case in the second part of the process consisting of: lifting the bucket filled with the soil, swinging the whole excavator with respect to vertical axis, lowering the bucket and discharging it. Next, the bucket is brought back to the excavation place again. Discussing these motions, one has to taking in to account dynamic effects. It should be also noted that mentioned motions are lasting approximately the same time as the digging process. It is then worthy to try to minimize the time needed for bringing the filled bucket to the discharge place, and back to the digging site. It is then the aim of the paper to present an optimal control of such a minimum time process. The paper deals with an optimum problem of positioning an excavator bucket along prescribed trajectory using minimum time. The paper is illustrated with numerical results giving some optimal trajectories.

KEYWORDS: excavation, optimization, control.

1. INTRODUCTION

A standard excavation process, on the construction side, can be seen as composed of two parts. The first one is the process of digging and filling the bucket with soil or other material. The second part of the operation consists in lifting (L) the filled bucket, swinging (S) it with respect to a vertical axis, stopping (S) it at the place where it should be unloaded, and discharged (D). The whole process is below defined as LSSD.

Since about twenty years, many attention has been paid to robotics application in the construction industry. However, most attention has been paid to the digging processes. Relatively large number of works in this fields were presented at the International Symposia on Robotics in Construction. Among others, Budny *et al.*[3] proposed an load-independent control of excavation process along a prescribed trajectory. The idea of the paper was to propose a control system free of a number of sensors mounted on the excavator attachment.

The present study is dealing with the LSSD process, namely with the problem of minimum optimal positioning of excavator bucket from the position where it is filled with the soil, ending the digging, to the place where it

should be discharged. There are several important reasons to undertake this research. They can be listed as follows:

- (1) LSSD is taking often as much time as digging process;
- (2) automation of LSSD would decrease the operator efforts, comparing with hand controlled motion of the bucket;
- (3) optimization of LSSD should decrease the time and/or energy needed for its realization.
- (4) automation of LSSD should decrease the probability of accidents, due to human errors.

It should be noted that available modern software and hardware, with their decreasing cost, make practical realizations of discussed positioning possible. The 3D visualization on the screen of a monitor would allow an operator to locate the bucket position. Then with a “push button” command, supported by an appropriate algorithm, would allow to automatic motion of the bucket along prescribed trajectory to discharging place.

The positioning and optimal control, have found also interest in pneumatic and hydraulic systems applied in machines for construction industry. Rachkov *et al.* are considering optimal control of a pneumatic manipulator with performance index imposed on energy con-

¹⁾ Insttue of Mechanised Construction and Rock Mining, Poland, 02-673 Warsaw, ul.Racjonalizacji 6/8, e-mail: mch@imbigs.org.pl

sumption. Shih et al. propose to apply sliding mode method, in positioning of a pneumatic cylinder with high speed solenoid valves.

Starting this work, it was essential to assume an appropriate algorithm based on a rigorous mathematical back ground for proposed trajectory optimization (TO). The first theoretical results in this field are due Pontryagin [5]. In his famous mathematical theory of optimal control the basic concepts of TO are presented. An application of the Pontryagin principle to a two arm manipulator is the work by Avetisian et al. [1].

Almost the same time with Pontryagin theory, emerged the non-linear programming (NLP) with its basic theorems by Kuhn-Tucker. Its development has been strongly dependent of digital computers, and then gave very powerful tool to solve many problems with discretized variables. In a survey paper by Betts [2] application of NLP and of some other methods related to TO are presented. Recently, Furukawa [4] proposed a trajectory planning discretizing functions entering in the problem, into piecewise constant functions. Roh and Kim [6] are proposing an indirect, applying time FEM method, combined with NLP. In the proposed algorithm is taking some elements of the both, last mentioned papers. The curve, joining initial and final bucket positions, is divided into a given number of equal elements. Unknowns in NLP problem are times needed to travel along a segment with a constant velocity. The system of equations and inequalities arising from NLP problem are solved by successive approximations method. In the first step, some independent variables are assumed, and other solved from state equations. Next, the remaining equations are solved from state conditions equations, giving values for next approximation. Some illustrative examples are presented at the end of the study.

2. KINEMATICS AND DYNAMICS

Consider a simplified model of an excavator with a loaded bucket of mass M_1 . Simplification consists in assumption that all three members of the excavator attachment constitute one solid beam of length $2L$ and mass M_2 . The arm, driven by a hydraulic actuator, can rotate with respect to a horizontal axis, by an angle α (Fig. 3). Additionally the arm, with the bucket,

can rotate with respect to a vertical axis by an angle φ . The latter motion is due a hydraulic motor rotating the carriage.

The considered system is then of two degrees of freedom. Its kinetic energy, a function of two unknowns angular velocities $\dot{\alpha}$ and $\dot{\varphi}$ is equal to:

$$T = \frac{1}{2} I (\dot{\alpha}^2 + \dot{\varphi}^2 \cos \alpha) \quad (1)$$

where:

$$I = 4 \left(M_1 + \frac{4}{3} M_2 \right) L^2$$

The kinetic energy reaches its value from work V exerted by external forces:

$$V = MgL \sin \alpha \quad (2)$$

where:

$$M = 2M_1 + M_2$$

From the second order Lagrange equations we get the following equations of motion:

$$\begin{aligned} I \left(\ddot{\alpha} + \frac{1}{2} \dot{\varphi}^2 \sin 2\alpha \right) &= Q_1 - MgL \cos \alpha \\ I \left(\ddot{\varphi} \cos^2 \alpha - \dot{\varphi} \dot{\alpha} \sin 2\alpha \right) &= Q_2 \end{aligned} \quad (3)$$

The system is then transferred in to a set of four equations of first order each, by assuming the following variables

$$[\alpha, \varphi, \dot{\alpha}, \dot{\varphi}] = [x_1, x_2, x_3, x_4] \quad (4)$$

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= \frac{1}{I} \left[-\frac{1}{2} I x_4^2 - MgL \cos x_1 + Q_1 \right] \\ \dot{x}_4 &= 2x_3 x_4 \operatorname{tg} x_1 + \frac{Q_2}{I \cos^2 x_1} \end{aligned} \quad (5)$$

3. PRE-SHAPED INPUTS FOR MINIMUM TIME CONTROL

Our strategy now is related to minimum time needed for the bucket to travel from an initial position α_0, φ_0 to a final position α_f, φ_f . We are then discussing a pre-shaped function for control input, however this is related to an open-loop control. Only after finding this function it would be possible to design a close loop system.

The discussed problem is stated as follows: Find torques Q_1 and Q_2 , driving the arm with respect to the horizontal axis, and with respect to vertical one, assuring the shortest time for the bucket to move from the initial to the final position along a given curve. The problem can be stated in terms of formulate in the form: Find the minimum time

$$t_f - t_0 \rightarrow \min \quad (6)$$

to move the bucket from initial point α_0, φ_0 and initial velocity $\dot{\alpha}_0 = \dot{\varphi}_0 = 0$, to the final position α_f, φ_f , and final velocity $\dot{\alpha}_f = \dot{\varphi}_f = 0$

if the motion of the bucket is defined by state equations (5) and torques are Q_1 and Q_2 are bounded as follows:

$$\begin{aligned} -Q_{1,0} &\leq Q_1 \leq Q_{1,0} \\ -Q_{2,0} &\leq Q_2 \leq Q_{2,0} \end{aligned} \quad (7)$$

The equations (5), (6) and (7) represent a nonlinear optimization problem, which can be solved numerically only. We start with discretization of variables entering into the problem. The given trajectory between the starting bucket position α_0, φ_0 and its final position α_f, φ_f is divided into j_0 elements. The traveling time $t_f - t_0$ is then divided in j_0 time intervals of $h = \frac{t_f - t_0}{j_0}$ each.

The state equations (5) after discretization take the following form:

$$\begin{aligned} &I \frac{x_{i,j+2} - 2x_{i,j+1} - x_{i,j}}{h^2} + \\ &-\frac{1}{2} I \sin(2x_{i,j}) \frac{(x_{i,j+1} - x_{i,j})^2}{h^2} + \\ &+ MgL \cos x_{i,j} = Q_1 \\ &I \cos(x_{i,j}) \frac{x_{i,j+2} - 2x_{i,j+1} + x_{i,j}}{h^2} + \\ &-I \sin(2x_{i,j}) \frac{(x_{i,j+1} - x_{i,j})(x_{i,j+1} - x_{i,j})}{h^2} = Q_2 \end{aligned} \quad (8)$$

The following algorithm is proposed to solve the discussed problem.

Step 1. Assume lower bound for h : $h_1 = 0$. In this case constraints (7) are not fulfilled as torques would be infinitely large.

Step 2. Assume upper bound for h : h_u , assuring that torques found from state equations (8) fulfil constraints (7).

Step 3. Take $h = \frac{h_1 + h_u}{2}$.

Step 4. Solve torques Q_1 and Q_2 from state equations (8) and verify (7).

If constraints (7) are fulfilled, substitute for $h_u = h$.

If constraints (7) are nor satisfied, substitute $h_l = h$.

Step 5. If $h_u - h_l \geq e$, where e is assumed admissible error, go to Step 3.

If $h_u - h_l < e$ then STOP.

4. EXAMPLES

Assume the following data for values entering into the problem:

$$M_1 = 440 [kg] \quad M_2 = 160 [kg]$$

$$M = 1040 [kg]$$

$$I = 6500 [kg \cdot m^2]$$

$$L = 1.500 [m]$$

$$-2000 \leq Q_1 \leq 2000 [N \cdot m]$$

$$-6000 \leq Q_2 \leq 6000 [N \cdot m]$$

For the data specified above, two distinct situations are considered.

Case 1: The initial bucket position is $x_{1,0} = -30^\circ$, $x_{2,0} = 0^\circ$

Its final position is $x_{1,f} = 60^\circ$, $x_{2,f} = 120^\circ$

At both positions, the bucket velocity is equal to zero. The assumed traveling curve, represented by angles α and φ are given in Fig. 4 and Fig. 5. The final result, showing relations between torques Q_1 , Q_2 and time are given in Fig. 6, Fig. 7.

Case 2: The initial bucket position is the same as in Case 1.

The assumed traveling curve, represented by angles α and φ are given in Fig. 8 and Fig. 9.

The final results, showing relation between torques Q_1 and Q_2 and time are given in Fig. 10 and Fig. 11.

5. CONCLUSIONS

An optimization problem for minimum traveling time for an excavator bucket, between along given trajectory is presented. Numerical results show significant differences, which may take place, between torques/time relations for two different trajectories.

The problem extended to a real excavator with three degrees of freedom, could be easily implemented into control of LSSD processes in serial manufactured excavators.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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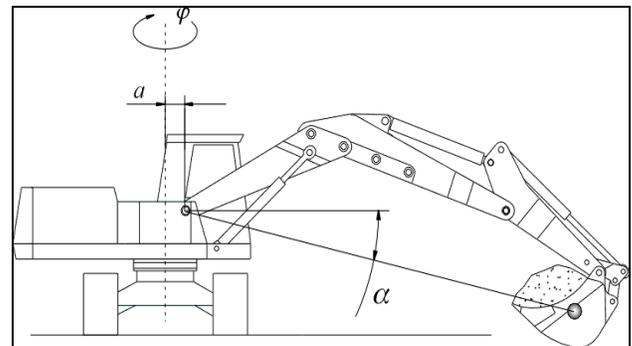


Fig. 1. Vertical projection of the excavator.

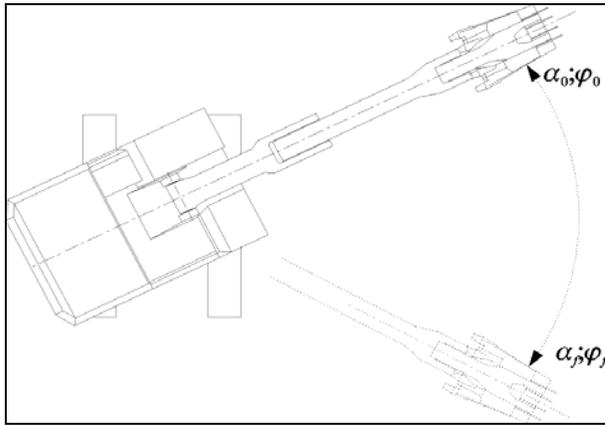


Fig. 2. Horizontal projection of the excavator

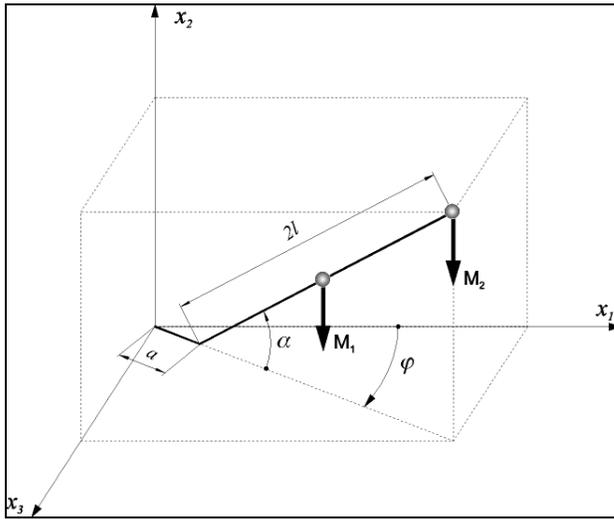


Fig. 3. Dynamic model of the excavator in coordinate system

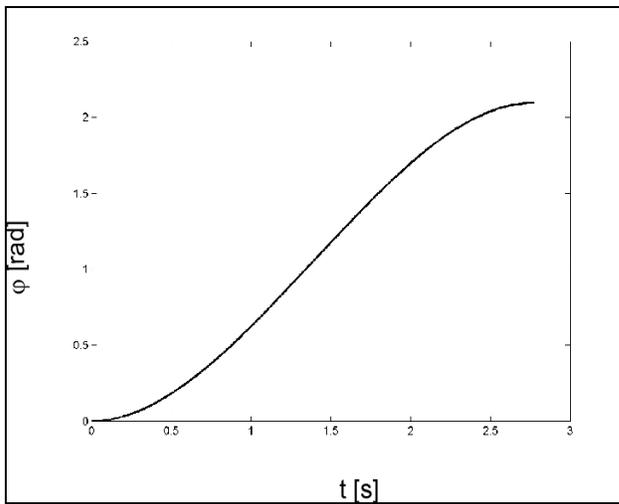


Fig. 4. Case 1: φ vs traveling time t

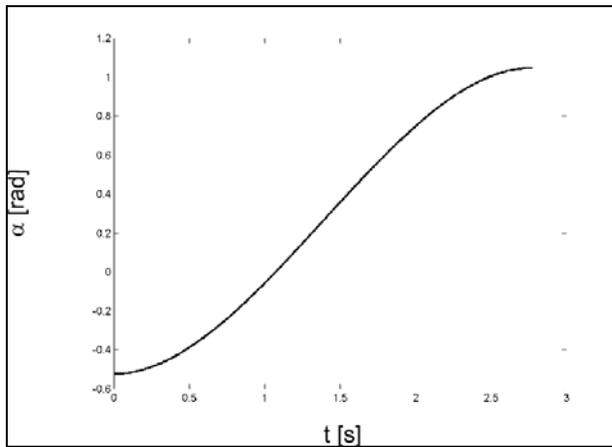


Fig. 5. Case 1: α vs traveling time t

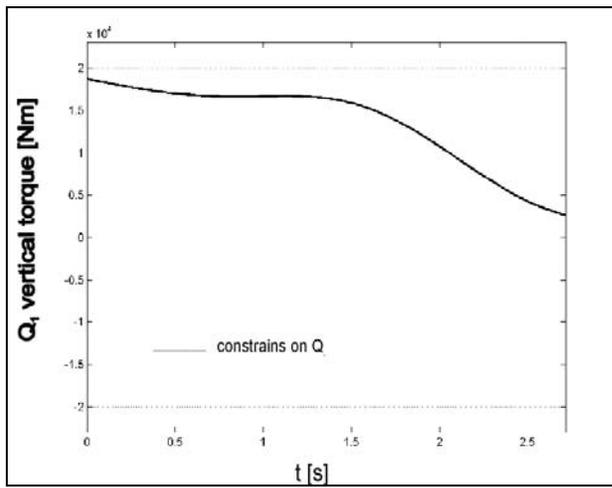


Fig. 6. Case 1: Vertical torque Q_1 vs traveling time t

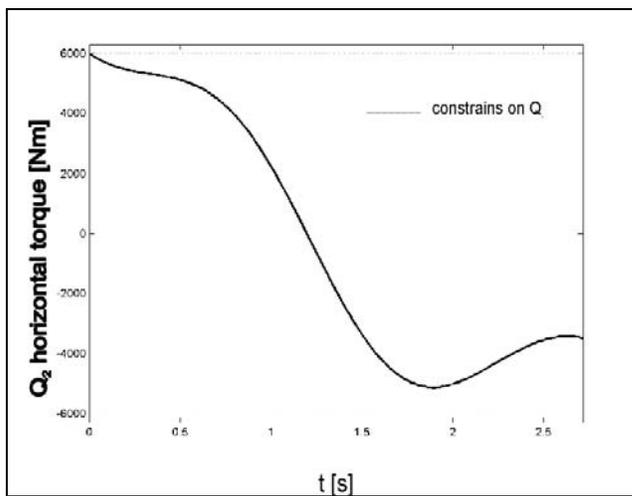


Fig. 7. Case 1: Horizontal torque Q_2 vs traveling time t

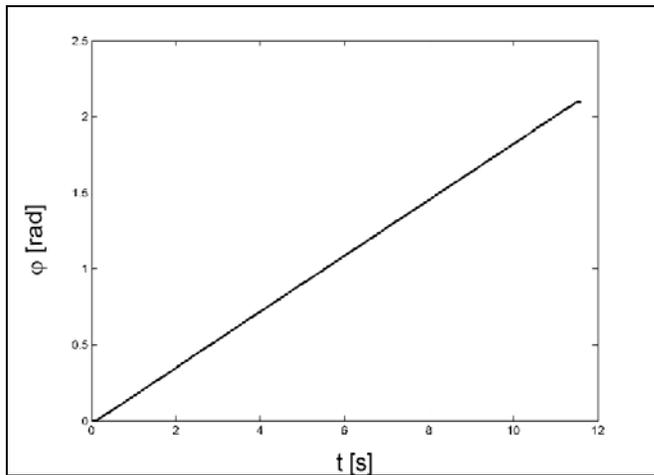


Fig. 8. Case 2: φ vs traveling time t

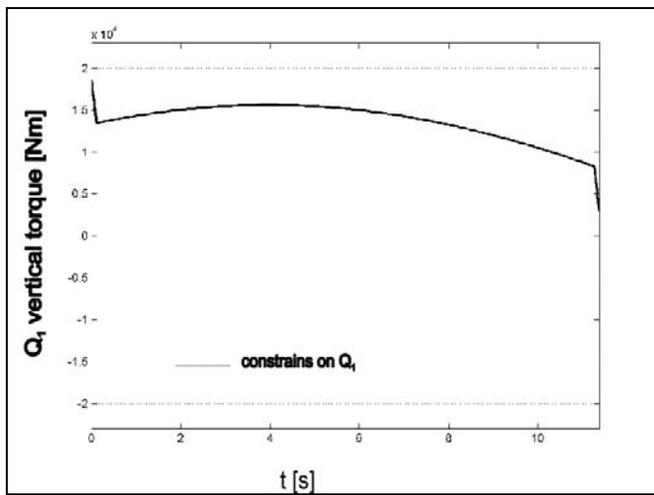


Fig. 9. Case 2: horizontal torque Q_1 vs traveling time t

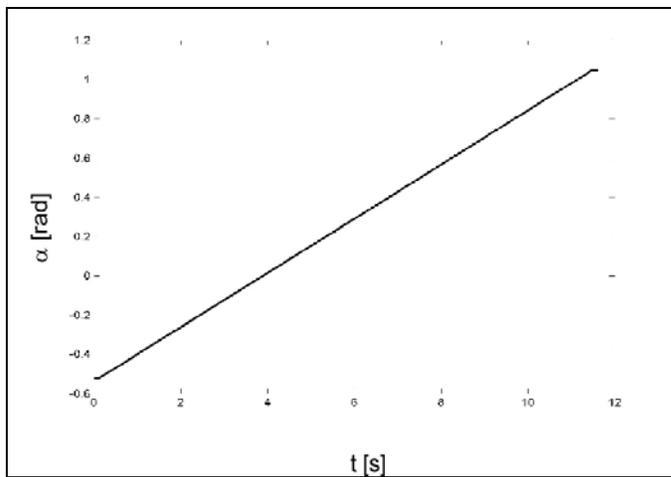


Fig. 10. Case 2: α vs traveling time t

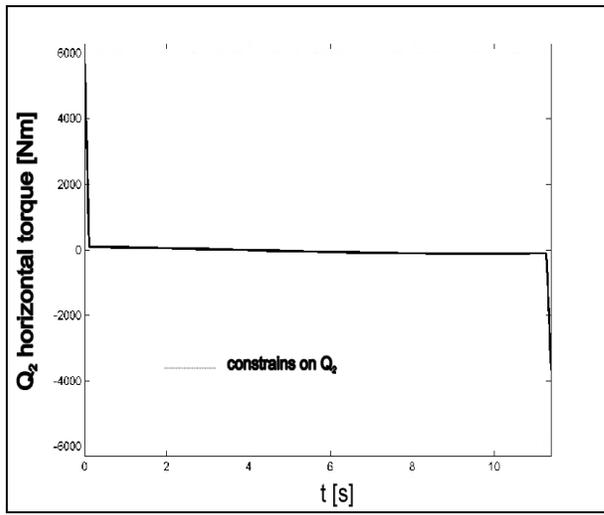


Fig. 11. Case 2: horizontal torque Q_2 vs traveling time t